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Anomaly characterization using Modified Wiener-Hopf filtering: a tool for mapping rock properties

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Summary

The design of a digital filter for using in specific geophysical application is very difficult task. In this paper, we report the development of an advanced digital filter which is robust and can map rock properties like susceptibility from the acquired magnetic data. To do this job, we modified the space domain Wiener-Hopf filter by introducing the scaling properties and make it capable to characterize source bodies from their produced magnetic anomalies. We furnish it as an effective tool for discriminating the geological feature of interest from the background noise. This modified filter was tested over the synthetic data sets with additive different Gaussian noise and was found to be working well. Afterwards, the filter is also applied on the real magnetic data acquired near Cuddapah basin and adjoining region to ensure the source body of the anomaly is due to lamprophyre/Kimberlitic or not. The application of this filter gave good result which is discussed in this paper.

Keywords: *Wiener-Hopf Equation, Error energy, Magnetic anomaly, Susceptibility*

Introduction

Anomaly characterization associated with potential field data is very ambiguous since the scale of the anomaly is not exactly same as the size of the anomaly. This is a case of non-uniqueness and therefore sometimes it is hard to differentiate and quantify the anomalous signature from the noisy signals (Negi et al., 1973; Dimri, 1992; Grant and West, 1965). However it is possible to interpret gravity data and structures prompting anomalous character by using eigen image analysis (Ganguli and Dimri, 2013).

The magnetic field is generally obtained by convolution of green function with susceptibility. The recovery of physical property of rock like susceptibility, density, etc. is possible by decolvolution of potential field data. It is seen that one of the mostly used optimum filter for potential field data is Wiener filter which minimizes the the mean squares error between the desired and the actual output. There are many application of Wiener filter for potential field data (Dimri, 1992; Gunn, 1972) like mapping of subsurface rock properties; reduction to pole; converting a magnetic to gravity as the pseudo gravity; thickness estimation of layers; etc. we focus on mapping the rock properties with the application of optimum filter.

The performance of Wiener filter is estimated by analyzing the error energy. For a single channel system, the error energy decreases with the increase in filter length and hence, the optimum lag Wiener filter is designed (Robinson and Treitel, 1980; Dimri, 1986). For a satisfactory analysis of magnetic susceptibility and to map rock properties accurately, a better technique is required. To improve the performance of the filter, we have imposed constraints on the Wiener-Hopf equation. Filter energy constraint is considered based on Backus-Gilbert formalism of linear inverse theory to improve the performance.

Analyzing Magnetic data gathered in volcanic provinces are often highly contaminated with noise originated, among others, by intrinsic uncertainties and magnetic heterogeneities. Regions where igneous rocks are predominate, usually exhibit very complicated magnetic variations.

Cuddapah basin of the southern Indian shield, has experienced subsequent plume activity during the

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Proterozoic and could well correspond to a 1.1 Ga kimberlitic activity. Mall et al., reported that Cuddapah basin is prone to magmatic activities from Proterozoic period. Anil Kumar et al. (2007) reported that the 1100 Ma kimberlite magmatism, was also proposed presence of a short lived mantle plume beneath the Dharwar region which coincides to a global period of ultra-potassic, alkaline and mafic magmatism. Thus, it is noteworthy to utter that the geo-anomalies (especially magnetic) associated with Cuddapah and adjoining areas suffered complex magmatism and reflects complex magnetic signals. To understand the nature of this magmatism and whether the source of the anomaly is from kimberlite pipes/Lamprophyres or not, the Modified Wiener-Hopf filter is used.

Methodology

The modified Wiener-Hopf normal equation is found to be very efficient and stabilized than normal Wiener-Hopf equation. Basically, the performance of the wiener filter has been improved by imposing the constraints of a well known Backus-Gilbert formalism of linear inverse theory. The two constraints are filter energy and the unimodular constraint. The former constraint helps in increasing the stability of Wiener filter, while the later one provides the unbiased estimation of the deconvolved output. We focus on filter energy constraint to design modified version of Wiener filtering. The derivation of modified Wiener-Hopf equation is due to Dimri (1978). Thus the filter energy constraint can be written as

$$\sigma_0^2 \sum_s \sum_t f(s,t)^2 = \text{constant} \qquad (1)$$

Where σ_0^2 is the variance of the noise which is assumed to be Gaussian.

Wiener filter is an optimum filter which minimizes the mean squares error between the desired and the actual output as

$$J = E\{d(a,b) - y(a,b)\}^2 \qquad (2)$$

where d(a,b) is the desired output; and y(a,b) is the actual output.

Since we are dealing with digitized signals, we convert Eq. (2) into discrete form as

$$J = \sum_{a=-(m+j)}^{(m+j)} \sum_{b=-(n+k)}^{(n+k)} \{d(a,b) - y(a,b)\}^2 \qquad (3)$$

and for a time-invariant system , the actual output can be considered to be connected with the input of the system as

$$y(a,b) = \sum_{s=-m}^{m} \sum_{t=-n}^{n} f(s,t) X(a-s, b-t) \qquad (4)$$

where J is the error energy ;X is the input of 2D array which consists of (2j+1)×(2k+1) elements; y has (2j+2m+1)×(2k+2n+1) elements; f(s,t) is filter coefficients or weighting function which has (2m+1)×(2n+1) elements; and d has (2j+2m+1)×(2k+2n+1) elements.

Substituting Eq. (4) in Eq. (3), we get

$$J = \sum_{a=-(m+j)}^{(m+j)} \sum_{b=-(n+k)}^{(n+k)} \{d(a,b) - \sum_{s=-m}^{m} \sum_{t=-n}^{n} f(s,t) X(a-s, b-t)\}^2 \qquad (5)$$

The aim is, therefore to determine the f(s, t) which converts the input into output for which Eq. (5) is a minimum in the least square sense.

Thus, differentiating Eq. (5) with respect to each filter coefficient, and equating the result to zero for getting minimum of it, we get

$$G_{dx}(i,j) = \sum_{s=-m}^{m} \sum_{t=-n}^{n} f(s,t) R_{xx}(i-s, j-t) \qquad (6)$$

where, R_{xx}(i-s,j-t) is the autocorrelation function of the input signal; and G_{dx}(i,j) is a cross-correlation between the desired output and the input signal.

Equation (6) is known as the Wiener-Hopf equation in the discrete form. It is also called Normal equation.

The modified Wiener-Hopf equation which is due to Dimri(1978), consider error energy as given by Eq. (4), is minimized subject to Eq. (1) following a method of Lagrange multiplier as

$$J = \sum_{a=-(m+j)}^{(m+j)} \sum_{b=-(n+k)}^{(n+k)} \{d(a,b) - \sum_{s=-m}^{m} \sum_{t=-n}^{n} f(s,t) X(a-s, b-t)\}^2 + \lambda \sigma_0^2 \sum_s \sum_t f(s,t)^2 \qquad (7)$$

where λ is the Lagrange multiplier.

To have the optimum filter coefficients, we differentiate Eq. (7) with respect to each filter coefficient and we the following set of equations are obtained:

$$G_{dx}(i,j) = \sum_{s=-m}^{m} \sum_{t=-n}^{n} f(s,t)\, R_{xx}(i-s, j-t) + \lambda\sigma_0^2\, \partial(i-s, j-t) \quad (8)$$

here, δ is the kronecker delta function and other term has the same meaning as described in Eq. (6). For λ=0, Eq. (8) reduces to Eq. (6) which is Wiener-Hopf equation.
Eq. (8) can also be written as

$$G_{dx} = F(R_{xx} + \lambda\sigma_0^2 I) \quad (9)$$

where I is identity matrix. The solution for Eq. (9) is found as

$$F = (R_{xx} + \lambda\sigma_0^2 I)^{-1}\, G_{dx} \quad (10)$$

R_{xx} is Toeplitz matrix like Wiener-Hopf equation, while the additional factor in Modified Wiener-Hopf equation is $\lambda\sigma_0^2$ which has increased all diagonal elements that increases the eigen values, thus avoiding the matrix from blowing up. The approach is similar to damped least square inversion.

The performance of a filter can be known by computing the normalized error energy which can be obtained as:

$$J = D_{dd}(0) - 2\sum_{s=-m}^{+m}\sum_{t=-n}^{+n}\{f(s,t)G_{dx}(i,j) + f^2(s,t)(R_{xx}(i-s,j-t) + \lambda\sigma_0^2\partial(i-s,j-t))\} \quad (11)$$

where $D_{dd}(0)$ is the zeroth lag autocorrelation function of the desired output. Using Eq. (8), Eq. (11) becomes

$$J_{\min} = D_{dd}(0) - \sum_{s=-m}^{m}\sum_{t=-n}^{n} f(s,t)\, G_{dx}(i,j) \quad (12)$$

here J_{min} is the minimum error energy and to find normalized error energy, we divide Eq. (12) by $D_{dd}(0)$, thus we get,

$$E = 1 - \frac{\sum_{s=-m}^{m}\sum_{t=-n}^{n} f(s,t)\, G_{dx}(i,j)}{D_{dd}(0)} \quad (13)$$

Forward Modeling

Forward modeling is done using the vertical field anomaly due to inclined unlimited prism or dike model (I.V.Radhakrishna Murthy, 1998) as

$$\Delta T_z = 2KFSin\theta[\cos Q(\alpha-\beta) - \sin Q \ln(r3/r4)] \quad (14)$$

where K is the susceptibility of the prism;F is the earth's magnetic field;Θ is dip and Q=(Θ-ϕ);ϕ is inclination of the prism.

Also, $\alpha - \beta = \tan^{-1}(r1/z) - \tan^{-1}(r2/z)$

here z is the depth of the prism from earth's surface; $r1 = [(X(i)-x)+w]$ and $r2 = [(X(i)-x)-w]$; $r3 = [z^2 + r1^2]$; $r4 = [z^2 + r2^2]$;w is the width; X(i) is the profile along which data is acquired and 'x' is the distance between the station and centre of the dyke.

We consider a dyke with the following input parameters: Z=450m; w=100m; Q=80^0, x=3000m K=0.55emu. Substituting these values in Eq. (14), we obtain a magnetic profile which is plotted in Figure (1).

In this particular problem, we consider input and desired output as 1D, whereas Eq. (10) & Eq. (13) have been derived for generalized case of 2D. Thus, for 1D version of these two equations, we consider the followings:

$$G_{dx}(i) = \sum_{s=-m}^{m} f(s)[R_{xx}(i-s) + \lambda\sigma_0^2\, \partial(i-s)] \quad (15)$$

$$E = 1 - \sum_{s=-m}^{m} f(s)\frac{G_{dx}(i)}{R_{dd}(0)} \quad (16)$$

Now, Eq. (15) & Eq. (16) can be used to estimate the filter coefficient series of an optimum filter and the error energy for different values of damping factor. The filter power is also computed. We have normalized the input and the desired output so that the trade-off curve can easily be obtained for selecting appropriate damping factor.

Response in presence of Noise

To check the feasibility of the Modified Wiener-Hopf filter, we have added Gaussian random noise associated with the magnetic profile which is obtained using Eq. (14). This is now considered as the input profile (Fig.2) to the optimum filter for better mapping of the susceptibility for which the error energy between desired and actual output will be minimum by means of Least square (L2 norm)manner.

Results

We prepared the trade-off curve (Fig.3) and from the inflection point we have seen that the optimum value of damping factor λ=1.001, which makes a compromise between the stability and accuracy of the filter. We then substituted the value of the damping parameter in Eq. (15) & Eq. (16) to get the optimum filter coefficients and these are then convolved with the input profile (Fig.2b) to get the actual output. The actual output is the susceptibility distribution which is shown along with the desired output as shown in Fig (4). It is note worthy to observe that the constrained filter gives better estimation of magnetization than the unconstrained filter in the presence of Gaussian random noise. We have also seen that the filter power is also reduced at the cost of small error energy for the constrained shaping filter (Fig 3).

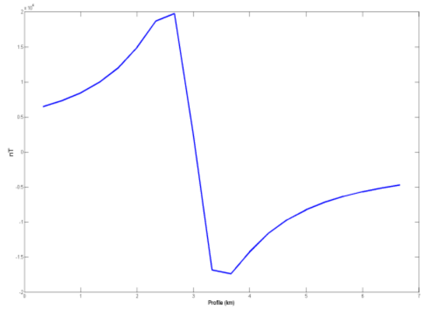

Figure 1: The desired output for a magnetic dyke.

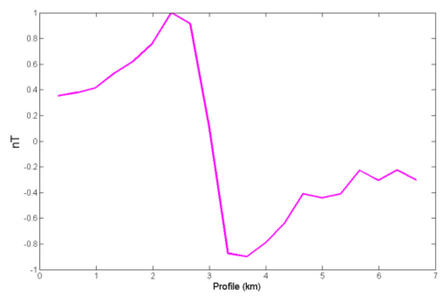

Figure 2: Magnetic profile added with gaussian random noise.

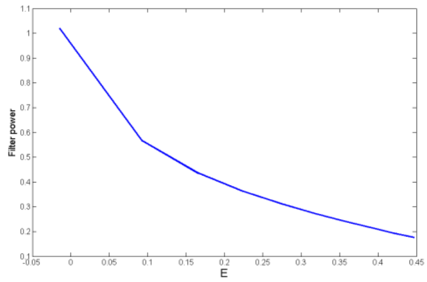

Figure 3: The trade-off curve.

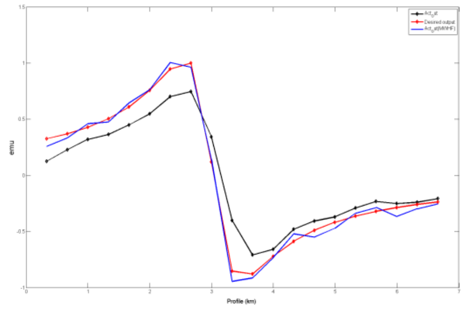

Figure 4: The actual filter output magnetization for both the filter along with desired output.

Field Example

We applied this tool with the aeromagnetic data collected over Cuddapah basin and adjoining areas, India. This basin has been believed to be one of the largest intra-cratonic proterozoic sedimentary basins of India situated in the eastern part of the Dharwar craton of the south Indian shield, which is magmatically infested and believe to be associated with diamond bearing kimberlitic pipe. The sole objective is to map the susceptibility of the region with this technique and to find the possibility of mapping those kimberlitic pipes. A magnetic profile from a 2D aeromagnetic map (Fig 5) acquired by CSIR-NGRI is considered and is shown in figure 6a.

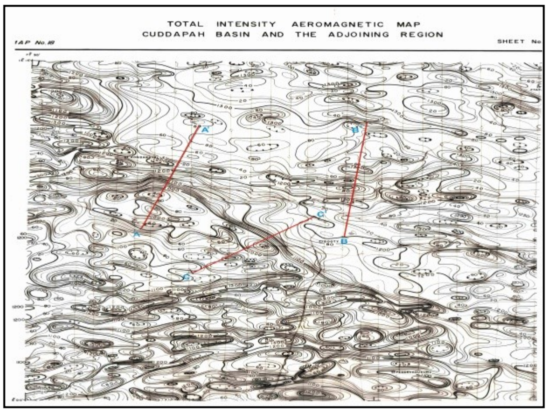

Figure 5: Aeromagnetic map of Cuddapah and adjoining area.

Field data have been digitized and the method described above has been implemented. We have constructed a model with two dyke swarms through areas A and B as shown in the figure 6b. We tried several values of susceptibility which is meaningful and accepted, later in our final modeling we assumed the magnetization for A is 0.11emu and for B is 0.08 emu. Following the normalization of the input profile and the desired output we obtain optimum value of the damping factor and also we found a good match between actual output (Susceptibility) and desired output with the model which we have considered (figure 6c).

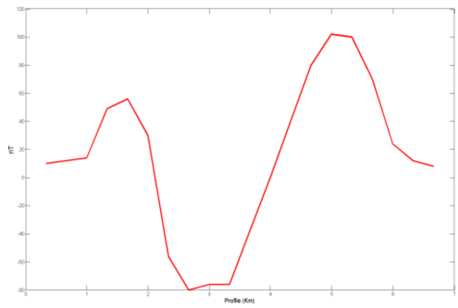

Figure 6a: The magnetic profile from the 2D aeromagnetic map.

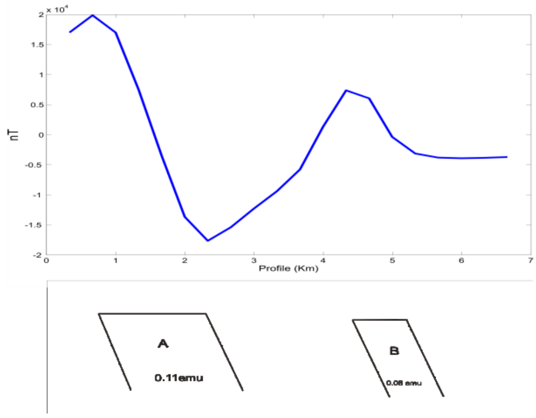

Figure 6b: The input model.

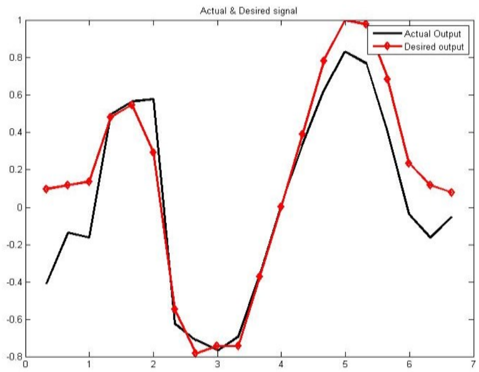

Figure 6c: The actual output and desired output susceptibility.

Conclusions

The Modified Wiener-Hopf filtering is very useful tool to decipher the susceptibility distribution of a region and it is found to be very effective and stable method than normal Wiener-Hopf filter. The constrained filter is actually stabilized due to consideration of the optimum value of the damping parameter and for this particular example the value is $\lambda=0.2$. Therefore, it is possible to characterize the magnetic anomaly accurately with this technique and it is also possible to map the rock properties efficiently.

The trade-off curve helps in finding the optimum value of damping factor λ, which makes a compromise between the stability and accuracy of the filter. It is very difficult to plot those curves for geo-potential data and hence, an alternative approach is proposed. It will be easy if we can normalize the input and the desired output and for this it is advisable to digitize the signal and divided by the largest value of it leading to a new set of data.

The profile considered in this particular problem can be interpreted as complex geological phenomena. The model with two consecutive dyke swarms shows a better match between the actual and desired signal. After rigorous trial, we obtain the susceptibility contrast for of one body is 0.11 emu and the other is 0.08 emu which shows a better agreement between desired and actual output. This could be correlated as a different source of magmatic intrusion and comparatively higher value of susceptibility relates to deep seated mantle plumes. If any such mantle plume ever existed, in all likelihood, such activities could invariably result into massive intrusion of carbonatites, kimberlites and lamprophyres.

It is worth noting that a significant susceptibility contrast of 0.11 emu for magmatic intrusions from the background indicates these are associated with mafic intrusions of short duration nature it supports Anil Kumar et al. (2007), they reported that the 1100 Ma kimberlite magmatism, suggests presence of a short lived mantle plume beneath the Dharwar region which coincides to a global period of ultra-potassic, alkaline and mafic magmatism. We couldn't conclude about whether the magmatism is associated with kimberlite pipes/Lamprophyres or not, to confirm it, a rigorous study of combination of various geophysical, geochemical analyses is needed. It is also mandatory to look into other profiles on the map before any final conclusions.

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