P 391

Frequency Dependent Reflection Coefficients in Anisotropic Media

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Summary

The reflectivity approach is followed to consider wave propagation in a multi-layered anisotropic medium. This approach enables us to investigate reflection coefficient as a function of angle, azimuth, frequency, and layer thickness. Plane wave propagation in homogeneous anisotropic media is solved by the Green-Christoffel equation. A stable formulation of layer matrix method for a multilayered medium is utilized for computational purposes. A layer matrix is a sextic form in which columns represent upgoing and downgoing waves for each wave types and rows represent Cartesian components of polarization and traction vectors.

In this work we have investigated frequency dependence of reflection coefficients. A layer of finite thickness sandwiched between two half spaces is considered as a model for our computations. The sandwiched layer has lower velocity compared to the two half spaces. Furthermore, thickness of the sandwiched layer is also changed to compute reflection coefficient for a monochromatic acoustic wave. Modeling of frequency dependent reflection coefficients is important to understand AVO effect in different frequency ranges which would essentially help in better interpretation of reflection seismic data.

Keywords*: Layer matrix, full waveform, reflection coefficients, frequency, anisotropy*

Introduction

A modified reflectivity method is used to consider wave propagation in a multilayered anisotropic medium. The continuity principle requires that the displacement and stress vectors be continuous throughout the medium, including the interfaces. Thus, there are two aspects of wave propagation in a multilayered medium: First, wave propagation in a homogeneous medium (i.e. within a layer) and second, wave propagation across an interface which gives rise to the reflection coefficients.

Most published papers treat reflection-refraction coefficients as an intermediate step in the course of computation of synthetic seismograms. A comprehensive review of various methods of this intermediary step can be found in the papers by Lowe (1995) and Rokhlin and Wang (2002). For wave propagation through a multilayered medium, the transfer matrix method was developed by Thomson (1950) and Haskell (1953). However, it was found to be computationally unstable for thick layers at high frequencies (Lowe, 1995). Though this method was developed for isotropic media, it was extended for general anisotropic media by Nayfeh (1991). Booth and Crampin (1983) extended the reflection matrix method to orthotropic symmetry, and Fryer and Frazer (1984) further extended it to include generally anisotropic layers. Rokhlin andWang (2002) developed a recursive stiffness matrix method in a manner similar to the reflection matrix method.

We have implemented the layer matrix formulation in a multilayered general anisotropic medium. A recursive method is developed to obtain the wavefield at any point in the multilayered medium. Apart from the wavefield determination, another outcome of this implementation of the layer matrix is a special purpose matrix representation of plane wave reflection and refraction coefficients associated with an interface. These reflection coefficients are computed at an interface of a multilayered medium to study the effects of frequency content of propagating waves and the thickness of a layer.

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Methodology

Suppose that the layer matrices for the layer 1 and 2 (Figure 1) are Lm_1 and Lm_2, respectively. Here, the layer 1 is a half-space and in the absence of any sources in the halfspace, one can safely assume that there are no downgoing waves in this layer. Let us represent the vectors of reflection-transmission coefficients ("R-T coefficients") in the upper-half space and the layer below as $(f\oplus \mid 0)^T$ and $(f'\oplus \mid f'\ominus)^T$. The superscripts $\oplus$ and $\ominus$ are for upgoing and downgoing waves, respectively. f is a vector of excitation coefficients. f and f' are excitation vectors in the layer 1 and layer 2, respectively. Since there is no downgoing wave in the upper half space, $f\ominus = 0$. Application of the continuity principle across the interface Z_1 gives:

$$Lm_1 \left(\frac{f^{\oplus}}{0}\right) = Lm_2 \left(\frac{f'^{\oplus}}{f'^{\ominus}}\right)$$

$$or,\ \ Lm_2^{-1} Lm_1 \left(\frac{f^{\oplus}}{0}\right) = \left(\frac{f'^{\oplus}}{f'^{\ominus}}\right) \tag{1}$$

By denoting $K = Lm_2^{-1}Lm_1$ in a block form, the above equation can be represented as:

$$\left(\frac{f'^{\oplus}}{f'^{\ominus}}\right) = \left(\begin{array}{c|c} K_{11} & K_{12} \\ \hline K_{21} & K_{22} \end{array}\right) \left(\frac{f^{\oplus}}{0}\right)$$

$$or,\ \ \left(\frac{f'^{\oplus}}{f'^{\ominus}}\right) = \left(\frac{\underbrace{K_{11} f^{\oplus}}}{\underbrace{K_{21}K_{11}^{-1}}\ \underbrace{K_{11} f^{\oplus}}}\right) \tag{2}$$

Equation 2 has the form $(F\oplus | K_r F\oplus)^T$ where $K_r = K_{21}K_{11}^{-1}$ and $F\oplus = K_{11} f\oplus$. This equation essentially translates the wavefield across an interface.

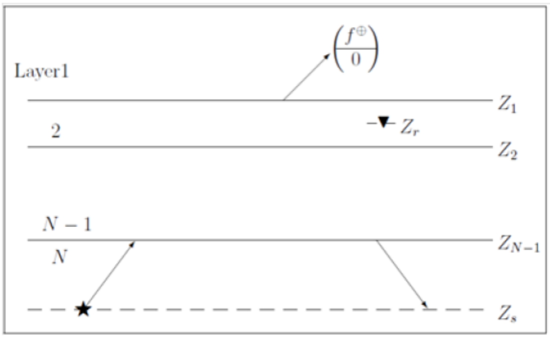

Figure 1: A schematic of the wavefield in a multilayered media above the source level *Zs*. Note that there is only upgoing wavefield in the upper half-space.

In the next step, let us consider the wavefield propagation in a homogeneous anisotropic layer. In this case, block matrix K has the following form

$$\left(\frac{f'^{\oplus}}{f'^{\ominus}}\right) = \left(\begin{array}{c|c} (E^{\oplus})^{-1} & 0 \\ \hline 0 & (E^{\ominus})^{-1} \end{array}\right) \left(\frac{F^{\oplus}}{K_r F^{\oplus}}\right)$$

or,

$$\left(\frac{f'^{\oplus}}{f'^{\ominus}}\right) = \left(\frac{\underbrace{(E^{\oplus})^{-1} F^{\oplus}}}{\underbrace{(E^{\ominus})^{-1} K_r E^{\oplus}}\ \underbrace{(E^{\oplus})^{-1} F^{\oplus}}}\right) \tag{3}$$

Once again, equation (3) has the form $(F^{\oplus} | K_r F^{\oplus})^T$ where $K_r = (E^{\ominus})^{-1} K_r E^{\oplus}$ and $F^{\oplus} = (E^{\oplus})^{-1} F^{\oplus}$.

Now let us look at the form of the wavefield in terms of R T coefficients when it propagates through the next interface. In this case the downgoing wavefield is non-zero. Therefore,

$$\left(\frac{f'^{\oplus}}{f'^{\ominus}}\right) = \left(\begin{array}{c|c} K_{11} & K_{12} \\ \hline K_{21} & K_{22} \end{array}\right) \left(\frac{F^{\oplus}}{K_r F^{\oplus}}\right)$$

or,

$$\left(\frac{f'^{\oplus}}{f'^{\ominus}}\right) = \left(\frac{\underbrace{(K_{11}+K_{12}K_r)F^{\oplus}}}{\underbrace{(K_{21}+K_{22})(K_{11}+K_{12}K_r)^{-1}}\underbrace{(K_{11}+K_{12}K_r)F^{\oplus}}}\right) \tag{4}$$

This is a general form of R-T coefficients across an interface. By using this equation over a loop, the wavefield across a stack of vertically inhomogeneous anisotropic layers is obtained. The form of Kr is equivalent to the frequency dependent reflection coefficients described by Kennett and Kerry (1979) and Fryer and Frazer (1984).

Model: A VTI layer between two VTI half spaces

A layer of finite thickness in a multilayer medium acts as a scatterer, thus, making the reflection coefficients frequency dependent. The frequencies ω which parameterize the reflection coefficients from a multilayered stack as the whole, are from the formula of monochromatic plane wave. The model shown in Figure 2 has a VTI layer sandwiched between two VTI half-spaces of same elastic constants. The elastic constants and densities of the layers are given in Table 1.

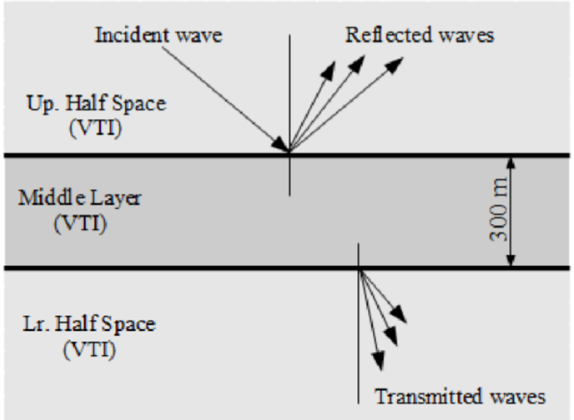

Figure 2: A schematic diagram showing a VTI layer sandwiched between two VTI half-spaces. The reflection coefficients are computed at the upper interface.

This model is investigated with the frequency ranging from 10 to 50 Hz and the reflection coefficients for the reflected P and S2 (SV) waves for an incident P wave are plotted in Figures 3 and 4. Since both the half-spaces are VTI with their symmetry axes perpendicular to the interface, there is no reflected S1 (SH) wave from the incident P-wave. The reflection coefficients and phase are colored by frequencies. Notice the change in PP reflection coefficients with the change in frequency. If the dominant frequency of investigation changes, the behavior of reflection coefficients with angle will change. Thus, it is important to include frequency effect in AVO interpretations.

C_s	Up. Half Space	Middle Layer	Lr. Half Space
C_{11}	59.4	21.26	59.4
C_{33}	42.42	17.63	42.42
C_{44}	15.27	15.32	15.27
C_{66}	19.7	8.99	19.7
C_{13}	15.82	6.97	15.82
ρ	2.37	2.34	2.37

Table 1: VTI elastic coefficients (in GPa) and density (in gm/cc) used to compute the reflection coefficients in the model Figure1.

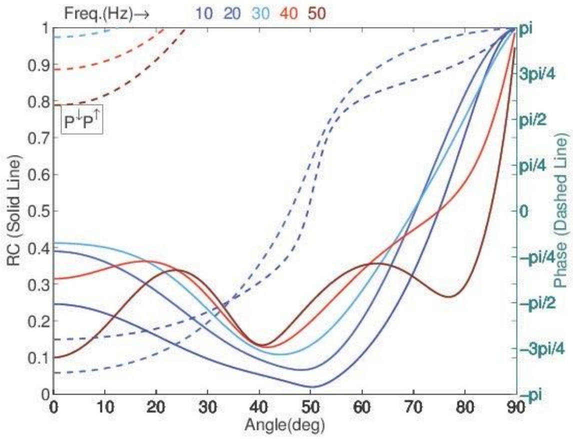

Figure 3: Reflection coefficient and phase for the reflected P-wave from an incident P-wave in the frequency range of 10 to 50Hz. The elastic coefficients for this calculation is given in Table 1.

Effect of thickness on reflection coefficients

Further, effects of the thickness variation on the reflection coefficients are analyzed. The model used for computation is similar to the model (Figure 2) used in the previous section, i.e. a VTI layer sandwiched between two VTI half spaces. The thickness of the sandwiched layer is varied from 210 to 300 m. The elastic coefficients and densities for the model are given in Table 1. The reflection coefficients are computed at 10Hz frequency for the reflected P and S2 (SV) waves from an incident P-wave as shown in Figure 5. Note that there is no S1 (SH) wave reflected for the incident P-wave in this model. It is evident from the plots that the thickness of a layer affects the reflection coefficients. Thus, AVO interpretation can be affected. Therefore, it is important to consider thickness variation in our models and compute reflection coefficients using exact formulations.

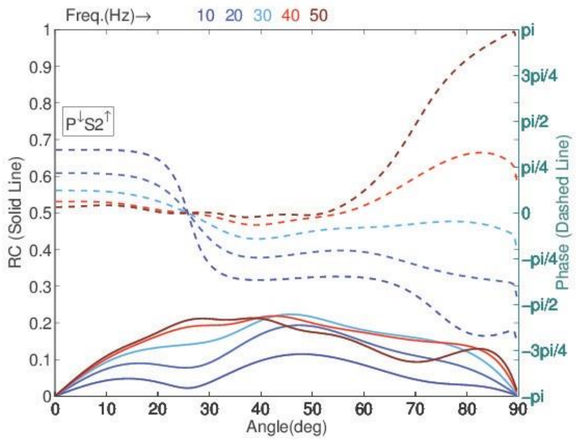

Figure 4: Reflection coefficient and phase for the reflected S2-wave from an incident P-wave in the frequency range of 10 to 50Hz. The elastic coefficients for this calculation is given in Table 1.

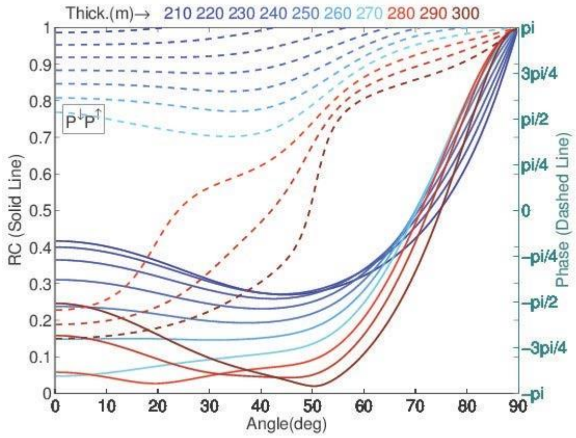

Figure 5: Reflection coefficient and phase for an incident P-wave and the reflected P-wave.. Thickness of the sandwiched layer varies from 210 to 300 m.

Summary

Reflection coefficient variation with frequency is modeled using reflectivity method. Layer matrix formulation has been made computationally stable in our numerical formulation. It helps in considering general anisotropic media. However, for simplicity we have modeled a VTI layer of finite thickness sandwiched between two half spaces. Frequency dependence of reflection coefficient is important to consider when we interpret AVO effect in reflection seismic data. Furthermore, thickness variation also affects reflection coefficient for different frequencies.

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