On elasticity of micro-inhomogeneous porous rock – The Brown and Korringa (1975) theory revisited

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Summary

Upon revisiting the Brown and Korringa theory we found that there is an internal inconsistency related to the underlying porosity changes associated with the compression of micro-inhomogeneous rocks. The origin of this inconsistency is the use of an incomplete differential in their derivation of the undrained bulk modulus. This inconsistency can result in unphysical predictions. Once this oversight is corrected, the resulting undrained bulk modulus coincides with the undrained bulk modulus obtained within the general poroelasticity framework. The results may have consequences for fluid substitution performed in fluid-saturated porous rocks with complex mineralogy and internal structure.

Introduction

Gassmann (1951) derives the undrained bulk modulus of a fluid-saturated, porous rock based on the assumption of a micro-homogeneous solid frame. This implies that under hydrostatic compression the solid frame deforms in a self-similar fashion and the porosity remains unchanged. Brown and Korringa (1975) generalize Gassmann's result to allow for micro-inhomogeneity. The associated porosity changes manifest in a difference between their unjacketed bulk and pore compressibilities. However, we find that the BK theory is not consistent with respect to the underlying change of porosity. Within the general poroelasticity framework there is yet another route to derive an undrained bulk modulus valid for the cases of micro-inhomogeneous solid and fluid phases (Sahay, 2013). Herein the change of porosity is explicitly quantified through the porosity perturbation equation. We verify that this framework is consistent with respect to porosity changes. The different expressions for the undrained bulk moduli are compared and contrasted.

Theory

More recently, Sahay (2013) deduces the undrained bulk modulus within a general poroelasticity framework.. It is (Sahay, 2013, equation 43; Müller and Sahay, 2013, equation 8)

$$K_{ud}^{*} = K_0 + \alpha\alpha^{*} M^{*} \tag{1}$$

where

$$\alpha^{*} = \alpha - (1-n)(\alpha - \eta_0) \ . \tag{2}$$

Herein η_0 is unperturbed porosity, α is the Biot coefficient

$$\alpha = 1 - \frac{K_0}{K_s}, \tag{3}$$

and the generalized specific fluid storage coefficient

$$\frac{1}{M^{*}} = \frac{\eta_0}{K_f} + \frac{\alpha^{*} - \eta_0}{K_s} \ . \tag{4}$$

K_s and K_f are the solid and fluid phase bulk modulus, respectively. K_0 is the drained bulk modulus. Equation 2 involves a new parameter *n* that can be viewed as the effective pressure coefficient for porosity. It is bounded by

$$0 \le n \le \frac{1}{\delta_{Ks}}, \text{ where } \delta_{Ks} = 1 - \frac{K_0}{(1-\eta_0)K_s}. \tag{5}$$

It is distinct from the term that goes by the same name in the context of the effective stress principle. It is a measure of how the pressure in a given phase alters if the pressure of the other phase is changed. When the porous medium is microhomogeneous, the deformational potential energy gets equalized over the representative volume element and there is one to one correspondence yielding *n* to be unity. When the porous medium is not homogeneous microscopically, there is localization of the deformational potential energy within the microinhomogeneity, which in turn makes *n* different than unity. If the deformational potential energy is getting partially localized in the interfacial region because of surface roughness (even though solid frame may be homogeneous) or within the bulk part of the solid because the frame may be multimineralic, *n* would be less that unity. If the process is such that there is not enough time for the fluid to equalize the deformational potential energy throughout its space, there shall be its localization in some part of the fluid space and *n* will be more than unity.

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If n=1 then $\alpha^* \to \alpha$ and the Gassmann result is obtained:

$$K_{ud} = K_0 + \alpha^2 M \,, \tag{6}$$

where $M^{-1} = \phi_0 / K_f + (\alpha - \phi_0)/K_s$ is the specific fluid storage coefficient of Biot's theory (Wang, 2000).

Equation 1 is consistent with the fundamental pressure equations as obtained from the mass balance equations of a biphasic solid-fluid system and the underpinning porosity perturbation equation of a linear, compressive deformation process. That is the change of porosity $\phi - \phi_0$ is

$$\phi - \phi_0 = -(1-\phi_0)\frac{\alpha - \phi_0}{K_0}(p^s - n\, p^f) \,, \tag{7}$$

where p^s and p^f are the macroscopic solid and fluid pressures. If n=1 we obtain the porosity perturbation equation implicit to Biot's theory (Sahay, 2013).

Müller and Sahay (2013) have shown that experimental data which do not obey Gassmann's equation can be well modelled using the Sahay (2013) framework.

Revisiting the Brown and Korringa theory

The change of porosity in the realm of the Brown and Korringa (BK) theory is governed by the differential and the fluid pressure, as becomes clear from the associated porosity perturbation equation (PPE). The latter follows directly from the definitions of the jacketed and unjacketed bulk and pore compressibilities (see, for example, Detournay and Cheng, 1993, equation 26b). However, the BK undrained bulk modulus can be also obtained by reconciling the pressure equations for the BK case (see for example Wang, 2000, equations 2.24 and 2.25) with a PPE that only depends on the differential pressure, as in the case of a micro-homogeneous rock. This is in contradiction of BK's anticipation of porosity changes in micro-inhomogeneous rocks and reveals the missing self-consistency in the BK theory. The origin of this inconsistency is the use of an incomplete differential in their analysis of the bulk volume changes. Specifically, their equation 7 is in error and the incurring error for the undrained bulk modulus will be analysed below.

Numerical comparison

For modelling purposes it is useful to express all quantities in terms of fundamental poroelastic parameters, namely the end-member properties K_s, K_f, δ_{Ks} and n. δ_{Ks}, defined earlier in (5), is related to Biot coefficient through

$$(1-\phi_0)\delta_{Ks} = \alpha - \phi_0 \,. \tag{8}$$

This allows δ_{Ks} to be viewed as a rescaled version of Biot coefficient. It spans from $0 \le \delta_{Ks} \le 1$, since α spans from ϕ_0 to unity.

Figure 1 shows a comparison of the BK undrained bulk modulus and equation 1 as a function of the porosity effective pressure coefficient. The example is computed for a water-saturated, porous rock with porosity $\phi_0 = 0.2$, $K_s = 35$ GPa and $K_f = 2.2$ GPa.

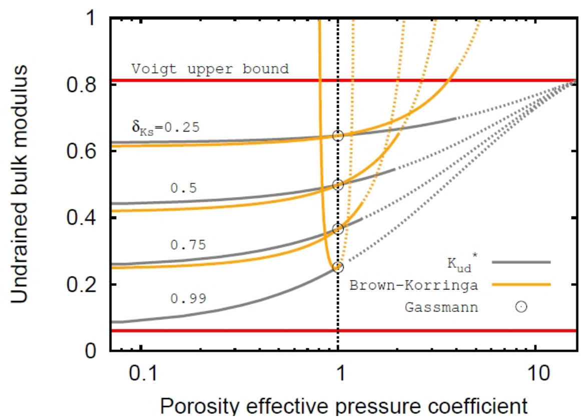

Figure 1: Undrained bulk modulus normalized by the mineral bulk modulus K_s as a function of the porosity effective pressure coefficient for varying δ_{Ks} parameter (gray: the undrained bulk modulus (equation 1), orange: Brown and Korringa theory, the circles denote the Gassmann prediction). Dashed lines are used for the regime where $\delta_{Ks} n \ge 1$.

We note that the BK undrained bulk modulus can become unphysical for certain (δ_{Ks}, n) combinations, as it exceeds the Voigt upper bound (see Figure 1, the curve labelled with δ_{Ks} =0.25 exceeds the Voigt upper bound for n > 3). In contrast, the undrained bulk modulus K^*_{ud} (equation 1) results in physically realizable predictions for the entire (₵$_{Ks}$, n) parameter space.

Conclusions

The Brown and Korringa (1975) theory is inconsistent with respect to porosity changes. This may lead to unphysical predictions for the undrained bulk modulus. On the contrary, the poroelasticity framework which entails the undrained bulk modulus as given by equation 1 is consistent and yields physically meaningful predictions in the whole parameter domain. It only involves one additional parameter over Gassmann's theory, namely the effective pressure coefficient for porosity (n) . It is a bounded parameter. Therefore, we recommend to use equation 1 for fluid substitution in porous rocks.

References

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