



Stochastic Seismic Inversion for Lithofacies using Rock Physics and Multiple-Point Geostatistics

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Summary

This paper presents a novel Bayesian inversion technique combining rock physics and multiple-point geostatistics (MPG) to predict lithofacies and saturations from seismic data. Rock-physics principles are incorporated at the beginning, defining the links between reservoir properties (e.g., lithology, saturation) and physical properties (e.g., compressibility, electrical conductivity). MPG is used to define and explore the space of solutions, in terms of geologically plausible spatial patterns of lithofacies. The proposed method can be applied in its current implementation to any inverse problem that can be approximated as a series of 1D forward-modeling operators. The method can be extended to satisfy multiple physical constraints simultaneously; in other words, the solutions can be conditioned with different types of geophysical data. Results of two synthetic tests and a real data application demonstrate the validity and applicability of the proposed inversion technique.

Introduction

Geophysical methods for reservoir characterization aims to reduce uncertainty in predictions of rocks and fluids away from wells and control points. The transformation of geophysical data, including seismic data, into reservoir properties is an inverse problem. However, in geophysical jargon, the definition of seismic impedance inversion is commonly restricted to the process of obtaining elastic properties in the subsurface from seismic data. Then, using rock physics and statistical techniques, each elastic property value is statistically classified into reservoir properties.

Different methods for inverting seismic data to elastic properties have been presented. In particular, the technique presented by Haas and Dubrule (1994) incorporates sequential Gaussian simulation for obtaining impedances from seismic data. In the context of the general stochastic inversion formalism (e.g. Tarantola, 2005), any geostatistical inversion can be seen as a technique of using geostatistics for defining and exploring the space of solutions. The objective of including geostatistical information in Monte-Carlo-based inversion algorithms is to give

priority to configurations of the model parameters consistent with particular spatial correlations. In the same line, inversion based on Monte-Carlo methods with two-point geostatistics and data from multiple geophysical methods have been presented (e.g. Bosch, 1999).

To overcome the limitations of two-point statistics (for modeling complex structures or curvilinear features) Guardiano and Srivastava (1993) present the ideas of training images and multiple-point geostatistics (MPG). The training image can be defined as a representation of the expected type of geologic variability in the area of study. The central idea of the MPG paradigm is to capture multiple-point statistics or patterns from the training image using a predefined arrangement of pixels (template) and to use those patterns as the building blocks for the stochastic realizations.

This paper presents a new inversion technique, which combines rock physics with MPG simulation. The method as described, works for any inverse problem that can be approximated as a series of 1D forward-modeling operators. In addition, the technique can be easily extended to invert data from different geophysical methods simultaneously (e.g. seismic and electrical). The solutions to



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the inversion problem provided by the technique are realizations of predefined *groups* in the subsurface. A *group* is defined as a set of rocks with common lithology, fluids or any other reservoir property of interest. Each group has an associated distribution of physical properties to which the geophysical data respond. Although the method has been developed for 2D (surface-depth, or x - z) acoustic problems, it can be extended to a more general, 3D, multi-physics case.

Although any MPG technique can be adapted without changing the core structure of the entire inversion algorithm, a modified version of the SIMPAT algorithm (SIMPAT*) was selected as the MPG component. SIMPAT, or sequential simulation with patterns (Arpat, 2005), as its name suggests, is a pattern-based geostatistical algorithm inspired by the problem of image reconstruction. The main modifications introduced to SIMPAT are the way of defining the random path and the cells that are filled when visiting any particular position of the grid.

Proposed algorithm for seismic inversion

The seismic inversion method is based on the Bayesian formulation of an inverse problem (e.g. Tarantola, 2005). Rock physics and MPG principles are used to constrain and explore the space of possible solutions. In addition to the innovative way of combining rock physics and MPG, a fundamental difference between this method and any other seismic inversion technique is that the solutions are realizations of spatial arrangements of groups (index variable) in the subsurface. The groups are the building blocks used to construct the solutions, gathering rocks with similar reservoir properties such as lithology and fluids. The technique consists of two main steps: pre-processing and the inversion itself.

Pre-processing step

The pre-processing step is formed by two procedures: definition of groups, and pattern database construction. The groups are specifically defined for the reservoir to be analyzed, according to the goals of the study. The distribution of each group's elastic properties is assumed multivariate Gaussian. The mean, variance, and covariance of the P-wave velocities (V_p) and densities (ρ) are derived from available well logs. Rock physics must be used to validate the wells observations and to extend them for predicting the elastic behavior of non-sampled groups expected to appear in the study area.

The pattern database is constructed by scanning the training image with a predefined template, which is moved one cell at each step, until all possible center pattern positions have been visited. Only unique patterns are retained in the database. Tran (1994) introduced the multiple-grid idea to account for large-scale structures or correlations, without increasing the number of cells in the template. For the inversion technique presented in this work, using a multiple grid means that for each grid level " $g+1$ ", only the (2^g)th grid position on the training image is considered. The values

of " g " start with zero, where all (consecutive) grid cells are accounted. For each grid level considered, a pattern database is generated. To increase the interaction between the grid levels, not only the selected pattern is pasted in the solution grid, but also some of the intervening cells skipped when defining the grid level. This idea of pasting intervening cells within the template was based on the dual-template concept proposed by Arpat (2005). Although the patterns always have the size of the template, the associated pattern size in the vertical direction changes with the grid level.

Inversion step: Seismic (normal incidence) inversion

The second step, the inversion itself, is an iterative process. Figure 1 shows a schematic representation of the procedures of a single iteration of the inversion step. First, a pseudo-random path for visiting all desired x positions is defined (the distance from the well position increases for each consecutive surface location selected). At every x visited, pseudo-logs of group indices for that location and its neighbors (inside a radius of half the template's horizontal length) are co-simulated with SIMPAT*. To generate the pseudo-logs, SIMPAT* first defines a random path for visiting all the z cells of the grid level in turn. At each z , the pattern from the database is selected that best matches (smallest Manhattan distance) the arrangement of group indices in the cells covered by the template. Then, its associated pattern is pasted in the solution grid, centered at the visited cell. Figure 2 illustrates the main SIMPAT* steps to populate the surrounding cells of a visited location. Particularly, Figure 2 shows the procedure corresponding to the first step, or the first visited cell (3-by-3 template, second level of grid). When the compared cells are hard data, the patterns that match exclusively those cells are selected first. The remaining cells covered by the template are considered for narrowing the choices that equally match the hard data. In the entire inversion process, the hard data is never changed.

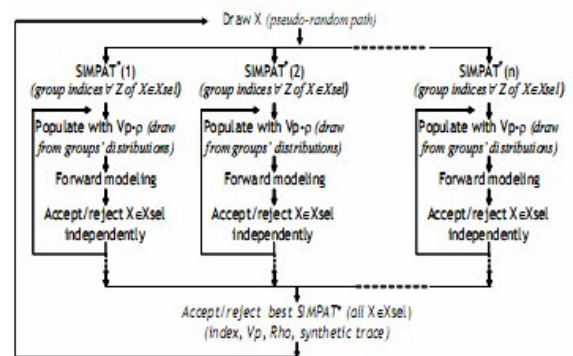


Figure 1: Schematic representation of the inversion step. An iteration is completed when all x (or CDP) positions defined in the pseudo-random path are visited.



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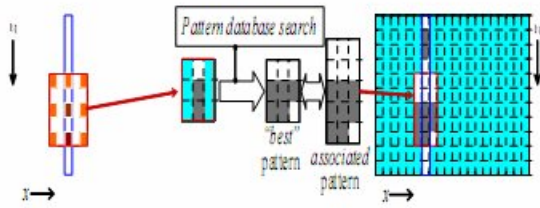


Figure 2: Main components of a single SIMPAT* step (step one in this case): selecting template cells to be compared ($g = 1$, i.e. second grid level), searching in the pattern database for the best match, and pasting the selected associated pattern.

For each pseudo-log of group indices generated with SIMPAT*, a user defined number of pseudo-logs of elastic properties are created drawing V_p and ρ values from the corresponding Gaussian distribution. A synthetic seismic trace is computed by convolving the reflectivity from each elastic property pseudo-log with a given (input data) wavelet. The synthetic traces that best match (L_1 -norm) the input data are selected (with the corresponding V_p and ρ pseudo-logs) as the result of each SIMPAT* simulation. Then, the quantity L (equation 1) is computed for every SIMPAT* realization (L_1, L_2, \dots, L_n).

$$L = \sum_{j=1}^{n_{tr}} \exp\left(-\sum_{i=1}^{n_{tr}} |syn_{xz}(j) - dat_{xz}(j)|\right) \quad (\text{Eq. 1})$$

The SIMPAT* realization with largest L (L_p) is selected as the proposed pseudo-logs. In case the visited x and its co-simulated neighbors were empty in the solution grid (before SIMPAT*), the proposed pseudo-logs are retained. On the other hand, if the simulated x locations were populated before, the best SIMPAT* realization is retained in the solution grid if L_p is greater than the L computed using the previous values of the solution grid. In case the proposed set of pseudo-logs is not accepted, the solution grid is transformed back to its previous state. An iteration ends when the pseudo-random path is completed, i.e. all x positions are visited, and the results are grids of group indices (accepted pseudo-logs), V_p and ρ , and synthetic traces computed. Many samples of the posterior probability function, i.e. many solutions or realizations, must to be analyzed to get a reasonable idea of the complete solution.

Synthetic test 1: Two groups

The two-dimensional models used in the synthetic tests were extracted from a modified version of the top layer of the Stanford VI synthetic reservoir, created by the geostatistics group in the department of

Petroleum Engineering at Stanford University. All the information about the model relevant to this work is shown in Figure 3. The model was a cross-section of a simplified, two-lithology channel system. It had 80 cells in z (total thickness of 80 m) and 150 cells in x or CDP (total length of 3750 m). Figure 3 also presents the spatial distribution of the V_p and ρ , a cross-plot between the two elastic parameters color-coded by the groups (channel-sand, background-shale), and the generated input seismic data (Kennett's algorithm with a 15 Hz Ricker wavelet). Notice the overlap between V_p - ρ points of the two groups. The training image used as input for the inversion was constructed by extracting twelve cross-sections from the modified Stanford VI synthetic reservoir, parallel to the one used to create the model. Although in the selected cross-sections the channels were a good representation of the channels in the true model, their spatial arrangements were different.

The values of the model (group indices, V_p , ρ) at CDP 40 and 120 were used as input data (wells) for the inversion. The mean, variance and covariance of V_p and ρ for each group were computed using only the provided wells. Thirty draws of elastic properties were completed for every pseudo-log of group indices simulated by SIMPAT*. Six iterations of the inversion step were completed to generate each realization or solution.

There are different ways to analyze the complete solution of the inversion. In exploration situations, the probability maps of each group, computed by averaging the group indicator at each cell through all realizations, are one of the best ways to visualize the solution. Probability maps can help to define the main geological bodies. On the other hand, for later stages of development of the area, each of the solutions (inversions after a defined number of iterations) needs to be considered, as in any stochastic method. The probability maps are a way to obtain an idea about the solution, but they are not the complete solution. It is always convenient to inspect some of the realizations individually, as well as the behavior of the value selected for measuring the misfit between the input data and the synthetic output (L in this case).

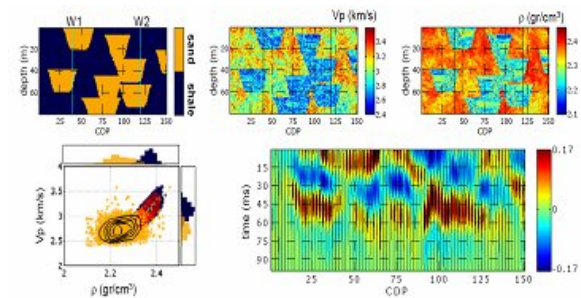


Figure 3: Model used for the first test (*two groups*): spatial distributions of group indices (top left), V_p (top center) and ρ (top right), plot of all of V_p and ρ values, color-coded by group (lower left), and the computed seismic data (lower right).



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The probability maps for sand and shale groups, obtained from 30 solutions (six iterations for each one) are shown in Figure 4. Sand channels, which coincide with those in the true model, can be clearly identified in the probability maps. As can be seen, in the deepest region of the probability maps (below 70 meters) between CDP 125 and 140, the shape of a small channel, which was not present in the group indices' input-model, can be discerned. A zone of relatively small elastic-property values, coincident with the referred area, can be seen in the input Vp and ρ sections, which, for this example, could well be attributed to the presence of sand channels.

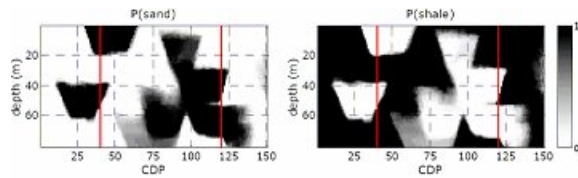


Figure 4: Probability map for sand (left) and shale (right) groups computed with 30 solutions (6 iterations for each one) of the proposed inversion (inverting seismic shown in Figure 3). Red vertical lines indicate the locations of the wells (CDP 40 and 120).

Synthetic test 2: Gas channels

Small changes to the geological framework were introduced for the second test. The channel located at a depth of 60 meters and CDPs 100 to 125 was disconnected from the others, changing some of its cells to shale. That channel, and the one centered at CDP 25 between depths of 40 and 60 meters were simulated to be gas saturated. The training image was generated based on the same 12 cross-sections used for the previous test. However, for each cross-section, 12 new images were generated by randomly assigning one of the fluid-lithology groups to the channels. During the process of generating the training image, the connectivity between channels was checked before assigning the saturating fluid in order to guarantee that any body was composed of a single group. Wells WA and WB did not sample any of the gas-saturated channels. From the information provided by WA and WB and the knowledge that there can be gas in the reservoir, the gas-sand group was defined. Its elastic properties were computed with the Vp and ρ values from the wells, using Gassmann's equations. The probability maps for each group computed with 10 solutions are presented in Figure 6. Even without sampling any of the gas-saturated channels, their correct locations were clearly defined by areas with high values in the gas-sand-group probability map.

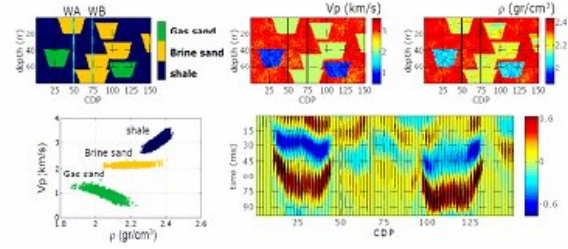


Figure 5: Model used for the second test (*gas channels*): spatial distributions of group indices (top left), Vp (top center) and ρ (top right), plot of all of Vp and ρ values, color-coded by group (lower left), and the computed seismic data (lower right).

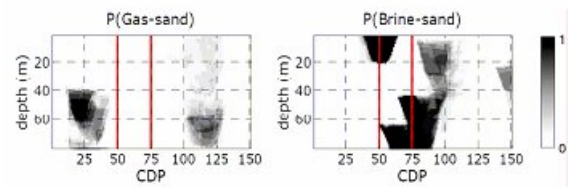


Figure 6: Probability map for gas-sand (left) and brine-sand (right) groups computed with 10 solutions of the proposed inversion (inverting seismic shown in Figure 5). Red vertical lines indicate the locations of the wells (CDP 50, 75).

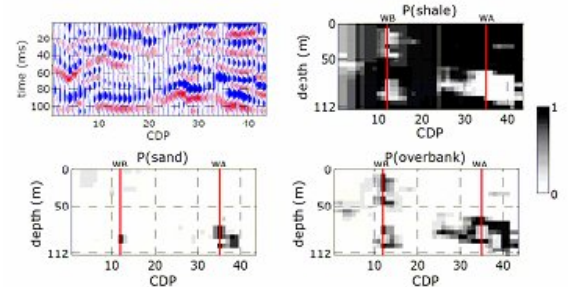


Figure 7: Input seismic data (top-left), probability map for shale (top-right), sand (bottom-left), and overbank (bottom-right) groups computed with 10 realizations of the proposed inversion.

Real data application

The proposed inversion technique works not only with synthetic data, but also in real situations. A 2D section extracted from a 3D volume was inverted. Three groups were defined based on the available two wells, training image and geological information: sand, overbank, and shale. The distributions of each group's Vp and ρ were obtained from the well-log data. Figure 7 presents the input seismic data and the probability for each group resulting from the inversion (10 solutions of 7 iterations each). Figure 7 demonstrates the applicability of the proposed inversion technique to real case studies.



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Conclusions

A novel inversion technique, which combines rock physics with multiple-point geostatistics, was presented. The method is based on the Bayesian formulation of an inverse problem, with rock physics and multiple-point geostatistics as the elements for constraining and exploring the space of possible solutions. The results of the tests presented validated the principles, the implementation, and the applicability to real situations of the inversion technique introduced. Probability maps for each defined group were used to summarize the results. In addition, multiple realizations optimally selected from the posterior distribution are also available for analysis. This inversion technique lays the foundation for innovative, multi-physics, multipoint inversions of geophysical data.

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