Introduction

Multiple attenuation may be classified in two main methodologies; 1) Prediction of multiples from the data itself, and 2) utilizing moveout separation between multiples and primaries. A commonly used version of the prediction approach is the so called SRME technique, where surface related multiples are predicted from the data itself, and at least in principle, does not require any further information. The SRME approach, certainly in its 3D implementation is very computationally intensive, but in recent years with the advent of commodity priced Linux clusters, has become very popular. However, the SRME technique does not work well in shallow marine environments and does not handle interbed multiples, thus there is still a need for approaches based on separation. In this paper we present the method utilizing multiple-primary separation in a tutorial fashion and show its progression from its simplest form in FK space to the latest time-domain Radon high-resolution demultiple.

Multiple attenuation by separation

The early methods utilizing moveout differences were applied in the F-K domain. The procedure was typically to first apply NMO with a velocity slightly slower than the medium velocity positioning primary events in the 2nd quadrant of the F-K domain while multiples which were still undercorrected were located in the 1st quadrant. After zeroing the 1st quadrant the data was inverse transformed. However, decomposing hyperbolic events as dip-components, which is in effect what the F-K transform achieves, does not give good separation, Nevertheless the method was used with some success until the mid 80s when Hampson (1986) showed how data could be decomposed efficiently into parabolas. Parabolas are much closer in shape to the hyperbolic shape of events in a cmp gather and after moveout with primary velocity the residual curvature in the data can be described by parabolas very well. In Hampson’s method the data is first transformed into the frequency domain and each frequency slice is transformed into the parabolic radon domain. Figure 2 shows the principle of the method, to the left is the data in the time-offset domain, the 2nd frame shows the Fourier transform with the vertical axis being frequency and the horizontal offset, the 3rd frame the parabolic transformation where the vertical axis is frequency and the horizontal curvature while the 4th frame shows the result after the inverse Fourier transform where the vertical axis is now time and the horizontal curvature.

The data in the time-curvature domain clearly shows how the 3 events have been decomposed into their parabolic curvature components and it is clear that the transform has decomposed the data into 3 distinct events which can be muted before the inverse transform. Hampson formulated the parabolic decomposition using least squares, finding the model representation that best match the original data after the inverse transform. In matrix vector notation this may be written as \( \mathbf{d} = \mathbf{Lm} \) where \( \mathbf{L} \) is a matrix representing the inverse Radon transform, \( \mathbf{d} \) is the data and \( \mathbf{m} \) the model representation. The forward least squares transform is given...
by: \( \hat{\mathbf{m}} = (\mathbf{L}^T \mathbf{L})^{-1} \mathbf{L}^T \mathbf{d} \). The matrix \( \mathbf{L} \) is only dependent on the data geometry and transform parameters, which typically do not change, and so can be pre-computed making this a very fast method. Although the Hampson method was a vast improvement over the F-K method, Figure 2, frames 3 and 4 demonstrate that events are not distinct and there is a substantial amount of smearing. As long as primaries and multiples are not overlapping this does not present a problem but in geologies where the moveout difference is small between primaries and multiples the smearing will present a problem. In the mid 90s Sacchi and Ulrych (1995) showed how a more high-resolution transform could be computed. The high-resolution Radon transform constrains the inversion by using the inverse of the model space, \( \mathbf{m} \), itself as a stabilizing matrix, \( \mathbf{S} \), in an iterative fashion:

\[
\hat{\mathbf{m}} = (\mathbf{L}^T \mathbf{L} + \mathbf{S})^{-1} \mathbf{L}^T \mathbf{d}
\]

matrix. Since \( \mathbf{m} \) is the model-space, and it occurs on the right hand side of the equation, it means an iterative scheme must be used.

Figure 3 demonstrates the high-res radon transform. Comparing the 3rd frames in figures 2 and 3 show that the high-resolution transform boosts high amplitudes and suppresses low amplitudes giving better separation in the transform domain.

In the example above there are only 3 events while the example in figure 4 shows 9 events. The 2nd frame shows the standard least squares frequency-curvature space. The nine events are now almost indistinguishable with energy over the entire curvature range making the job of boosting high and suppressing low amplitudes much harder. The 3rd frame of figure 4 shows the curvature-time domain and it is clear that the events in this domain are much better separated, so the idea behind the time-domain high-resolution radon transform is to apply the constraints in this domain rather than the frequency domain. Figure 5 shows the result using the time-domain constraints, the 9 events are now well separated and it would be easy to mute some of them before doing the inverse transform.

Field data example

In Figures 6 and 7 a field data example is shown. The input data has strong aliased multiples, which are a problem for the LS transform. The frequency domain HR transform performs significantly better, but still leaves some aliased energy in the data. The time domain transform removes virtually all aliased multiple energy. The stack is quite effective at attenuating the aliased multiple, but the time domain Radon transform is still visibly better than the frequency domain methods (see Figure 7).
Conclusions

Multiple attenuation using techniques that rely on the move-out differences between multiples and primaries are more effective as the smearing of events in the transform domain is reduced. From the first F-K methods to the Radon transform methods the objective has always been to obtain better separation. The time-domain high-resolution Radon transform presented in this paper shows very good separation and can be a very effective multiple attenuation tool.

References

Sacchi, M.D., and Ulrych, T.J., 1995, High-resolution velocity gathers and offset space reconstruction, Geophysics, 60, 4, 1169-1117.