Seismic Modeling and Amplitude versus Azimuth in Fractured Media

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Summary
Most fractures are too small to be resolved in the seismic frequency band. However, they affect seismic amplitude, arrival time and waveform in a systematic manner depending on the nature of the fracture pattern. Effective media theories are often used to represent a fractured medium which show that a fractured medium is equivalent to a homogeneous anisotropic medium whose symmetry depends on the fracture parameters. Occurrence of multiple fractures is fairly common and explaining the behavior of seismic data in such media poses challenging problems. Here I review some fundamental issues related to the representation of stiffness tensor of fractured media and their mapping to matrices. I will then present a case study from Canadian thrust belt and show how an unusual amplitude variation with azimuth can be modeled using multiple intersecting fractures.

Introduction
Many hydrocarbon reservoirs may have significant porosity but low permeability (for example, tight gas sand and coal bed). However, such reservoirs are often naturally fractured. The fracture patterns in these reservoirs can control flow and transport properties, and therefore, play an important role in drilling the production wells. Recognition of fractures or fracture networks in a hydrocarbon reservoir could be the key to its economic viability. Literature is abound with examples where understanding of fracture patterns helped save the potential reservoirs from being abandoned (e.g., Aguilera (1995); Nelson (2001)). Sometimes, the hydrocarbon recovery is completely dependent on the exploitation of the natural fracture networks in a reservoir. A high density of naturally occurring fractures has been recognized as a controlling factor for commercial success of production wells (e.g., Neves et al., 2003). Coal has negligible matrix porosity and permeability. Reservoir gases are not contained in granular pores, instead they are absorbed on the surface of the coal. Although coal has impermeable matrix, it has an anisotropic fabric of natural fractures known as cleats. The cleat system is composed of two orthogonal sets of fractures known as the face cleats and the butt cleats (e.g., Laubach et al., 1998). Exploitation of this cleat system is crucial for commercial production of gas from the coalbed methane reservoirs. Nelson (2001) lists the following reasons to understand the fracture patterns in a reservoir: (1) Early assessment of reservoir’s potential, (2) Optimization of well locations and paths, (3) Accurate prediction of field rates and recovery, and (4) Economic depletion of field.

There are six main fracture parameters which might be of relevance to a reservoir geo-scientist or engineer: (1) Fracture orientation, (2) Fracture spacing, (3) Fracture aperture, (4) Fracture area, (5) Fracture porosity, and (6) Fracture morphology. It is not possible to obtain detailed estimates of all of these fracture parameters since seismic data only reveal some amplitude and phase anomaly that can be used to estimate some (or combination of) these attributes. In the seismic frequency band, closely spaced parallel fractures behave like an anisotropic medium. A number of equivalent media theories were proposed in the last two decades. The most general and accurate equivalent elastic coefficients for cracks and fractures. This observation suggests that the shape of the fractures cannot be uniquely resolved by seismic methods. The equivalent elastic coefficients are expressed in terms of fracture parameters such as fracture orientation, fracture or crack
density and fracture infill (gas or fluid). The effects of fractures in seismic data are manifested in seismic data in the form of shear wave splitting and P-wave amplitude variations with offset and azimuth (AVOA). Here I outline a brief review of anisotropic behavior of a fractured medium with examples of seismic response from complex fracture patterns such as non-orthogonal intersecting fractures.

Theory

**Stiffness and Compliance**

We will follow cartesian axis system for the development reported in this paper. We assume linear elasticity given by the following constitutive equation

\[ \tau_{ij} = \sigma_{ijkl} \varepsilon_{kl} \]

together with an equation of motion to describe wave propagation in a general anisotropic elastic. In eq (1) the term \( \sigma_{ijkl} \) represents a fourth rank elastic coefficient tensor (also called a stiffness tensor), \( \tau_{ij} \) is a second rank stress tensor and \( \varepsilon_{kl} \) is a second rank strain tensor. One other equivalent form relates strain to stress tensors using the following equation

\[ \varepsilon_{kl} = (S_{ijkl})^{-1} \]

where \( S_{ijkl} \) is a fourth rank compliance tensor and is related to the stiffness tensor as follows

\[ S_{ijkl} = (\sigma_{ijkl})^{-1} \]

Note here that the inversion of a fourth rank tensor is not a trivial task, a point that I will elaborate further.

The tensor equation given in Eq. (1) is often replaced by a matrix-vector equation using a condensed notation exploiting the symmetry properties of the stiffness, stress and strain tensors. Without using any symmetry properties of the tensors, Eq. (1) can be rewritten as

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{23} \\
\sigma_{31}
\end{bmatrix} =
\begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} & \sqrt{\sigma_{11}} & \sqrt{\sigma_{12}} & \sqrt{\sigma_{13}} \\
\sigma_{12} & \sigma_{22} & \sigma_{23} & \sqrt{\sigma_{22}} & \sqrt{\sigma_{23}} & \sqrt{\sigma_{22}} \\
\sigma_{13} & \sigma_{23} & \sigma_{33} & \sqrt{\sigma_{33}} & \sqrt{\sigma_{33}} & \sqrt{\sigma_{33}} \\
\sigma_{11} & \sqrt{\sigma_{11}} & \sqrt{\sigma_{12}} & \sqrt{\sigma_{13}} & \sqrt{\sigma_{22}} & \sqrt{\sigma_{23}} \\
\sigma_{22} & \sqrt{\sigma_{22}} & \sqrt{\sigma_{23}} & \sqrt{\sigma_{33}} & \sqrt{\sigma_{22}} & \sqrt{\sigma_{23}} \\
\sigma_{33} & \sqrt{\sigma_{33}} & \sqrt{\sigma_{33}} & \sqrt{\sigma_{33}} & \sqrt{\sigma_{23}} & \sqrt{\sigma_{33}}
\end{bmatrix}
\]

(5)

One other representation of Eq. (4) is

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{23} \\
\sigma_{31}
\end{bmatrix} =
\begin{bmatrix}
\sigma_{11} & \sqrt{\sigma_{11}} & \sqrt{\sigma_{12}} & \sqrt{\sigma_{13}} & \sqrt{\sigma_{22}} & \sqrt{\sigma_{23}} \\
\sqrt{\sigma_{11}} & \sqrt{\sigma_{22}} & \sqrt{\sigma_{23}} & \sqrt{\sigma_{33}} & \sqrt{\sigma_{22}} & \sqrt{\sigma_{23}} \\
\sqrt{\sigma_{12}} & \sqrt{\sigma_{23}} & \sqrt{\sigma_{33}} & \sqrt{\sigma_{33}} & \sqrt{\sigma_{23}} & \sqrt{\sigma_{33}} \\
\sqrt{\sigma_{13}} & \sqrt{\sigma_{23}} & \sqrt{\sigma_{33}} & \sqrt{\sigma_{33}} & \sqrt{\sigma_{23}} & \sqrt{\sigma_{33}} \\
\sqrt{\sigma_{22}} & \sqrt{\sigma_{23}} & \sqrt{\sigma_{33}} & \sqrt{\sigma_{33}} & \sqrt{\sigma_{23}} & \sqrt{\sigma_{33}} \\
\sqrt{\sigma_{23}} & \sqrt{\sigma_{23}} & \sqrt{\sigma_{33}} & \sqrt{\sigma_{33}} & \sqrt{\sigma_{23}} & \sqrt{\sigma_{33}}
\end{bmatrix}
\]

(6)

The systems (5) and (6) are entirely equivalent. However, there is a significant difference in that the 6X6 matrix in Eq. (6) represents components of a second rank tensor in six dimensions and the matrix in Eq. (5) does not. This imposes serious implication in inverting a fourth rank stiffness tensor into a fourth rank compliance tensor. This cannot, in general, be achieved by inverting the 6X6 stiffness matrix.

Using Voigt’s condensed notation Eq. (5) is written as

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{23} \\
\sigma_{31}
\end{bmatrix} =
\begin{bmatrix}
C_{1111} & C_{1122} & C_{1133} & \sqrt{C_{1111}} & \sqrt{C_{1122}} & \sqrt{C_{1133}} \\
C_{1122} & C_{2222} & C_{2233} & \sqrt{C_{2222}} & \sqrt{C_{2233}} & \sqrt{C_{2233}} \\
C_{1133} & C_{2233} & C_{3333} & \sqrt{C_{3333}} & \sqrt{C_{3333}} & \sqrt{C_{3333}} \\
C_{1111} & \sqrt{C_{1111}} & \sqrt{C_{1122}} & \sqrt{C_{1133}} & \sqrt{C_{2222}} & \sqrt{C_{2233}} \\
C_{1122} & \sqrt{C_{1122}} & \sqrt{C_{2222}} & \sqrt{C_{2233}} & \sqrt{C_{2233}} & \sqrt{C_{2233}} \\
C_{1133} & \sqrt{C_{1133}} & \sqrt{C_{2233}} & \sqrt{C_{2233}} & \sqrt{C_{2233}} & \sqrt{C_{2233}}
\end{bmatrix}
\]

(7)

Similar equivalent expression can be written down for Eq. (6). Note that although manipulating Equations (5), (6) and (7) is very convenient, there is a fundamental problem when one wants to map from stiffness to compliance and vice versa. Eq. (7) is most commonly used in seismology and engineering applications. Note, however, that a straightforward inversion of stiffness matrix by matrix inversion does not yield its equivalent compliance matrix. Neadeau and Ferrai (1998) have shown that the compliance matrix that yields an equivalent 6X6 stiffness matrix equivalent to the one used in Eq. (7) is as follows

\[
S = \begin{bmatrix}
S_{11} & S_{12} & S_{13} & 2S_{12} & 2S_{13} & 2S_{11} \\
S_{12} & S_{22} & S_{23} & 2S_{22} & 2S_{23} & 2S_{22} \\
S_{13} & S_{23} & S_{33} & 2S_{23} & 2S_{33} & 2S_{33} \\
2S_{12} & 2S_{22} & 2S_{33} & 4S_{12} & 4S_{22} & 4S_{33} \\
S_{13} & 2S_{23} & 2S_{33} & 4S_{13} & 4S_{23} & 4S_{33} \\
2S_{13} & 2S_{23} & 2S_{33} & 4S_{13} & 4S_{23} & 4S_{33}
\end{bmatrix}
\]

(3)

Thus we note that the compliance and stiffness tensors map differently to comply with meaningful mapping of tensors to matrices. The system given by Eq. (6) is a more convenient option but is not generally used in seismology.
Fracture to Stiffness mapping: Equivalent Media Theory

Micro fractures, pores and other heterogeneities with uniform statistical distribution can be replaced by an equivalent or effective medium provided that the scale of the observation is much larger than the scale of heterogeneities. The replacement of the inhomogeneous media with an equivalent continuum might make the material anisotropic. In exploration seismology, this approach is favored since a highly heterogeneous isotropic medium can be replaced with a homogeneous anisotropic medium with fewer parameters. Such an approximation is generally valid since the predominant seismic wavelength is much larger than the heterogeneities present in the subsurface.

The most widely accepted equivalent media theories for fractured rocks were given by Hudson (1980) and Schoenberg (1980) for penny-shaped cracks and parallel planar discontinuities, respectively. Hudson (1980) modeled a fractured medium as a distribution of penny-shaped cracks for which a scattering integral is evaluated analytically assuming low-frequency wave propagation and an equivalent stiffness matrix is given in terms of the fracture parameters. On the other hand Schoenberg (1980) and Schoenberg and Sayers (1995) used a linear slip model to derive equivalent anisotropic coefficients of a fractured medium. It is possible to relate Schoenberg parameters to Hudson fracture parameters. Schoenberg and Sayers (1995) showed that the elastic compliance tensor of a rock containing multiple fracture sets relates the average strain over a representative volume \( V \) to the average stress components. The effective strain is given by

\[
\mathbf{e}_{kl} = S_{ijkl}^b \sigma_{kl} + \sum_f S_{ijkl}^f \sigma_{kl},
\]

where \( S_{ijkl}^f \) is the excess compliance tensor due to a fracture set \( f \) which is related to the fracture parameters. Note here that we need to be consistent here with mapping of this 4th rank tensor to a 6x6 matrix as given in Eq. (8).

For a rotationally invariant fracture, this results in the following matrix

\[
S_f = \begin{bmatrix}
Z_N & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & Z_T & 0 & 0 \\
0 & 0 & 0 & 0 & Z_T & 0 \\
0 & 0 & 0 & 0 & 0 & Z_T \\
\end{bmatrix},
\]

where \( Z_N \) and \( Z_T \) are called the normal and tangential compliances. These are related to dimensionless fracture weaknesses \( \Delta_N \) and \( \Delta_T \) as follows

\[
\Delta_N = \frac{K_b Z_N}{1 + K_b Z_N}, \quad \Delta_T = \frac{\mu_b Z_T}{1 + \mu_b Z_T}.
\]

The values of \( \Delta_N \) and \( \Delta_T \) in terms of the fracture parameters are given in table 1. An algorithm for generating equivalent stiffness matrix for a set of non-orthogonal vertical fractures is displayed in Figure 1.

P-wave amplitude variation with offset and azimuth (AVOA)

Amplitude versus offset (AVO) analysis is a routine procedure for lithology estimation and direct detection of hydrocarbons. The primary assumption in such an analysis is that variation of reflection amplitude as a function of offset is due to changes in the reflection coefficient as a function of offset, or angle. Assuming that pre-processing has corrected for amplitude variations (losses) due to geometrical spreading, attenuation, anisotropy and lateral heterogeneity, the following fundamental assumptions are employed to analyze AVO data:

- A ‘primaries only P-wave reflection’ model,
- Reflections are from an isolated interface, and
- Transmission effects are negligible.

An exact reflection coefficient for such an idealized model is given by the so-called Zoeppritz (1919) equations. These expressions are complicated and do not impart a clear understanding of how reflection coefficients are affected by changes in rock properties. A popular simplification introduced by Shuey (1985) assumes a small angle of incidence and weak contrast between rock properties across a welded interface. This expression has been written in many forms, but for the purposes of this paper we choose to express it as follows:

\[
R_{av} (\theta) \approx A + B \sin^2 (\theta) + C \tan^2 (\theta),
\]

where \( A \) is referred to as the AVO intercept, \( B \) the AVO gradient and \( C \) represents a curvature term. This equation is valid for isotropic media and artificially imposes a cylindrical symmetry. Subsequently, reflection amplitudes are only dependant on the angle of incidence and not azimuth.

Using static equivalent media theory it can be shown that a vertically fractured material is elastically equivalent to a transversely isotropic medium with a horizontal axis of symmetry (HTI). An expression similar to Shuey’s equation is derived by Ruger (1998) to account for
azimuthal dependence of amplitude in HTI media. In Ruger's derivation the AVO gradient term consists of two parts, an azimuthally independent (isotropic) term, and a second anisotropic term that is azimuth dependent. The resulting equation is

\[ B(\phi) = B_{iso} + B_{ani} \cos^2(\phi - \phi_{sym}) , \]

where \( \phi \) is the source-receiver azimuth, and \( \phi_{sym} \) is the symmetry axis orientation, or the strike of the fracture. The final linearized reflection coefficient is given by

\[ R_{lin}(\theta, \phi) = A + [B_{iso} + B_{ani} \cos^2(\phi - \phi_{sym})] \sin^2(\theta) . \]

The above equation is a commonly employed amplitude variation with offset and azimuth (AVOA) equation. A more general equation for linearized reflection coefficient for a general anisotropic medium has been derived by Shaw and Sen (2004) and has been applied in direct detection of fluid (Shaw and Sen 2006) and estimation of dip of fractures (Shaw et al. 2007).

Some New Observations and Modeling

Note that the usual AVOA equation for a single vertical fracture predicts a simple sinusoidal variation of P-wave amplitude with azimuth. Recently Sen et al. (2007) reported on some AVOA observation over fractured reservoir that showed asymmetric AVOA pattern as shown in Figure 2. Several attempts were made to predict these observations using single vertical and dipping fracture set with or without fluid. In all cases, the predicted AVOA was found to be symmetric. Finally synthetic seismograms were computed using reflectivity and finite difference seismograms (Bansal and Sen 2007) for an isotropic layer over an anisotropic half space. Two anisotropy models were found to be particularly interesting. The first consisted of two non-orthogonal fluid filled vertical fracture sets and the second consisted of two vertical non-orthogonal dry fracture sets. It appears that the AVOA predicted by the first model (non-orthogonal fluid filled vertical fracture) is still symmetric (Figure 3a) while that predicted by the second model (non-orthogonal dry vertical fracture) is asymmetric (Figure 3b) which resembles the field observations.

Discussions and Conclusions

In this paper we reviewed some fundamental principles of constructing stiffness and compliance tensors for fractured medium and demonstrated the need for incorporating complex fracture pattern and full wave modeling in understanding amplitude variation with azimuth. The recipe for constructing these elastic coefficients is outlined in detail in a flow diagram. Care must be taken in mapping the 4th rank stiffness and compliance tensors to their equivalent reduced matrix forms such that the mapping honors tensor inversion.

Acknowledgements

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References

(CBM) reservoirs, Journal of Seismic Exploration, Submitted.


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<tr>
<th></th>
<th>$\Delta_N$</th>
<th>$\Delta_T$</th>
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<tbody>
<tr>
<td>Fluid-filled</td>
<td>0</td>
<td>$\frac{16\epsilon}{3(1 - 2\gamma)}$</td>
</tr>
<tr>
<td>Gas-filled (dry)</td>
<td>$\frac{4\epsilon}{3(1 - \gamma)}$</td>
<td>$\frac{16\epsilon}{3(1 - 2\gamma)}$</td>
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Table 1: Fracture weaknesses for dry and fluid filled fractures are given in terms of fracture parameters such as fracture density and $\gamma = \frac{\nu}{\sigma}$, the ratio of shear wave velocity to P-wave velocity of a background isotropic medium.
Figure 1: A general Flow diagram describing a procedure for generating a stiffness matrix of a medium consisting of multiple fracture sets.

Figure 2. Asymmetric gradient (far offset – near offset amplitude) versus azimuth data observed over a fractured reservoir in the Lynx field Canada (from Sen et al. 2007).
Figure 3: Amplitude variation with azimuth for an incidence angle of 20 degrees for a model consisting an isotropic layer over a fractured (anisotropic) half space. The top panel is for a model comprising two intersecting fluid filled vertical fractures while the lower panel is for a model comprising two intersecting dry vertical fractures. Note that the dry fractures show asymmetric AVA pattern.