P-25

WAVELETS : A Mathematical Microscope

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Summary:

Geophysical Seismic signal Processing (GSSP) is of paramount importance for imaging underground geological structures and is being used all over the world to search for petroleum deposits and to probe the deeper portions of the earth. Expanding the frequency bandwidth of surface seismic data is an unending quest for geophysicists, because increased seismic resolution is essential for extracting stratigraphic detail from seismic images. While both vertical resolution and horizontal resolution are important for interpreting small geologic features on seismic data. Deconvolution for temporal/ vertical resolution while migration spatial/areal horizontal resolution. If the frequency spectrum of a seismic wavelet is centered around 30 Hz, which is usually achievable, and the seismic interval velocity is greater than 3000 m/s, reservoirs having a thickness less than 25 meters may not be resolved. "Not resolved" means there is no distinct reflection peak or trough centered on the top and bottom interfaces of the reservoir unit. This interval thickness, where seismic data can no longer position a distinct reflection peak or trough at the top and base of the interval, is called "tuning thickness. "Because numerous stratigraphic targets have thicknesses of 10 meters or less – which is thinner than tuning thickness for most seismic profiles – frequency enhancement procedures need to be applied to seismic data to study reservoir targets in this "thinner than tuning thickness" domain. Seismic Signals are Statistical in nature with non-stationary character. A new technique signal processing called wavelet processing has been developed to help obtain the better resolution for the detection of thin layer and to provide improved data for stratigraphic interpretation. The objective of wavelet processing is to optimize the shape of the seismic pulse and make it a symmetrical or zero-phase wavelet, which is the simplest form and the one the Interpreters desire.

Wavelet Transform Teager-Kaiser (WT-KE) Energy – A Seismic Attribute Applied to Reveal Geological Features. A new method to estimate the instantaneous seismic traces energy is presented here. We propose to use the Teager-Kaiser energy associated with wavelet transform to generate a joint time-frequency representation, which can be used as a nonlinear energy tracking of the seismic waves.

Introduction

Wavelets provide an alternative approach to traditional signal processing techniques such as Fourier analysis for breaking a signal up into its constituent parts. The driving impetus behind wavelet analysis is their property of being localised in time (space) as well as scale (frequency). This provides a time-scale map of a signal, enabling the extraction of features that vary in time. This makes wavelets an ideal tool for analysing signals of a transient or non-stationary nature(mono-component & multi-component nonstationary signals). Standard DWT (Discrete Wavelet Transform), being non-redundant, is a very powerful tool for many non-stationary Signal Processing applications, but it suffers from three major limitations; 1) shift sensitivity, 2) poor directionality, and 3) absence of phase information.

To reduce these limitations, many researchers developed real-valued extensions to the standard DWT such as WP (Wavelet Packet Transform), and SWT (Stationary Wavelet Transform). These extensions are highly redundant and computationally intensive.

Complex Wavelet Transform (CWT) is also an alternate, complex-valued extension to the standard DWT. Complex wavelet transform with excellent directionality, reduced shift sensitivity and explicit phase information

Earth is low pass filter which is inevitable constraint for high frequencies source signals.Nevertheless high

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frequency imaging is done with the help of wavelet transform of low frequencies output signal recorded in reflection exploration seismology.

Recently, the world of two dimensional transforms has been considerably expanded by the introduction of new varieties of two dimensional multiresolution transforms. Examples of them are ridgelets, beamlets, ridgelet packets, curvelets,diplet and contourlets.

Theory :

The Wavelet Transform

A wavelet is a continuous time signal that satisfies the following properties

$$\int_{-\infty}^{\infty} \psi(t)dt = 0$$

$$\int_{-\infty}^{\infty} |\psi(t)|^2 dt < \infty$$

where $\psi(t)$ is defined as the mother wavelet

The continuous wavelet transform

$$W(a,b) = \int_{-\infty}^{\infty} y(t)\psi^*_{a,b}(t)dt$$

where y(t) is any square integrable function,a is the dilation parameter, b is the translation

parameter and $\psi^*_{a,b}(t)$ is the dilation and translation(* asterik denotes the complex

conjugate) of the mother wavelet defined as

$$\psi^*_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right)$$

Scaling, as a mathematical operation, either dilates or compresses a signal. Larger scales correspond to dilated (or stretched out) signals and small scales correspond to compressed signals.

In terms of mathematical functions, if f(t) is a given function f(st) corresponds to a contracted (compressed) version of f(t) if s > 1 and to an expanded (dilated) version of f(t) if s < 1 . However, in the definition of the wavelet transform, the scaling term is used in the denominator, and therefore, the opposite of the above statements holds, i.e., scales s > 1 dilates the signals whereas scales s < 1 , compresses the signal

Time-Frequency Representation

Higher frequencies are better resolved in time, and lower frequencies are better resolved in frequency. This means that, a certain high frequency component can be located better in time (with less relative error) than a low frequency component. On the contrary, a low frequency component can be located better in frequency compared to high frequency component.

When we plot time-domain signals, we obtain a time-amplitude representation of the signal. This representation is not always the best representation of the signal for most signal processing related applications. In many cases, the most distinguished information is hidden in the frequency content of the signal. The frequency SPECTRUM of a signal is basically the frequency components (spectral components) of that signal. The frequency spectrum of a signal shows what frequencies exist in the signal.

Uncertainty Principle", which states that, we cannot exactly know what frequency exists at what time instance , but we can only know what frequency bands exist at what time intervals.Joint time-frequency analysis technique is employed to resolve uncertaintity constraints of signals. FT gives what frequency components (spectral components) exist in the signal. When the time localization of the spectral components are needed, a transform giving the Time-Frequency representation of the signal is needed.
Fourier transform to windowed short time Fourier Transform (STFT) , multiresolution analysis (MRA)- analyzes the signal at different frequencies with different resolutions ,Gabor and wavelet transform, time and frequency analysis, phase plane representations, Why wavelets: Localization by wavelets, decorrelation by wavelets, transient nonstationary data analysis, time-frequency localization, signal separation

The seismic wavelet is normally assumed to be minimum phase (i.e. the energy is "front loaded"). This assumption is allowable because: Impulsive seismic sources produce wavelets which are reasonable approximations to a minimum phase wavelet. ,It can be shown that most of the processes causing distortion are also minimum phase ; Convolving minimum phase with minimum phase produces a minimum phase output. This assumption is useful because the deconvolution operator may be conveniently designed in thefrequency domain . However to do this we need information about both the amplitude spectrum and phase of the seismic wavelet.

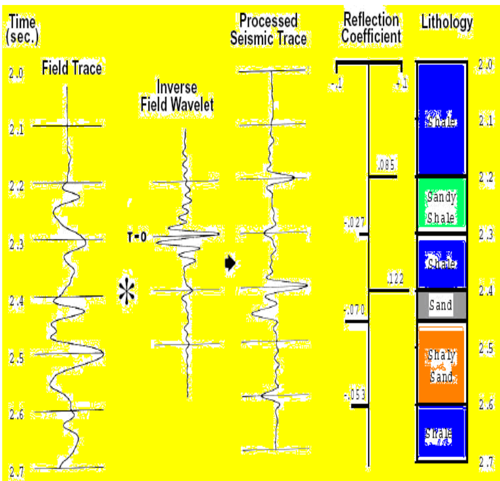


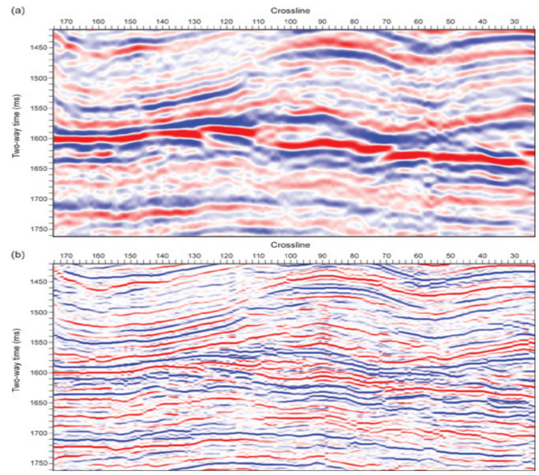
Fig.1: When the field wavelet is known, deterministic deconvolution is able to produce a processed trace that contains the desired broad band - zero phase wavelet. Note, the highest amplitude in the processed

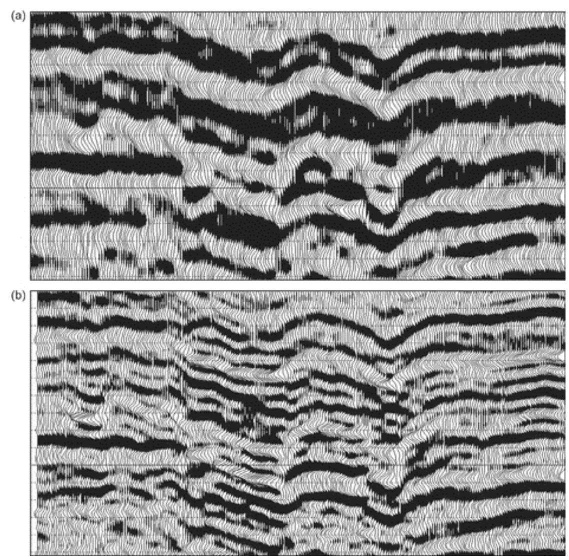
For data with a high signal-to-noise ratio, units with thicknesses less than the tuning thickness of the input data can be resolved.

The improved-resolution seismic data retrieved in the form of reflectivity data are not only important for more accurate geologic interpretations but prove to be advantageous for: Convolving the extracted reflectivity with a wider bandpass wavelet (say 5-120 Hz) to provide a high-frequency section.

Providing high-frequency attributes that enhance lateral resolution of geologic features.

Figure (2) – Comparison of (a) a conventional seismic section and (b) its derived thin-bed reflectivity. More geologic detail can be seen with the reflectivity data than with the input data.



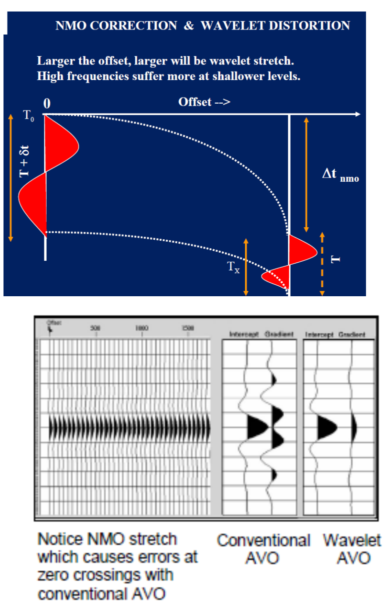


Figure(3) – Comparison of (a) a segment of a band-limited seismic section and (b) the equivalent section derived when thin-bed reflectivity is convolved with a 5- to 120-Hz bandpass wavelet. The section in panel(b) has enhanced resolution.

Wavelet-based AVO(Amplitude Variation with Offset) analysis(WAVO) :

Wavelet stretching due to NMO correction of seismic gathers causes problems in AVO. Coupled with the degrading action of wavelet stretching during NMO correction is offset dependent tuning for thin beds. Even though tuning is inherent in the data before NMO correction, its effect on AVO is more obvious on NMO corrected data. Studies have been carried out for an analytical understanding of NMO stretching and thin-bed tuning and their correction to improve AVO fidelity. Based on these studies, we have implemented the NMO stretching and thin-bed tuning corrections in a practical fashion for production

AVO analysis. Both synthetic and real data examples show that these corrections are necessary for performing reliable AVO analysis. The AVO (amplitude variation with offset) technique assesses variations in seismic reflection amplitude with changes in distance between shot points and receivers. AVO analysis allows geophysicists to better assess reservoir rock properties, including porosity, density, lithology and fluid content.



Figure(4): Presentation on NMO stretch

In the search for solutions to address increasingly complex reservoir challenges, E&P companies push the demand for more sophisticated technology. AVO - Amplitudes Versus Offset - is an example of a technology maturing to meet these increasing demands. The AVO now being analyzed in non-traditional geologic settings, such as basins with compacted rocks and in the deepest part of younger basins, is a more complex type of AVO. A strong focus on improving AVO technologies combined with a greater understanding of the effects of fracturing and anisotropy has produced the next wave in technology advancement - a wavelet based approach to AVO analysis.

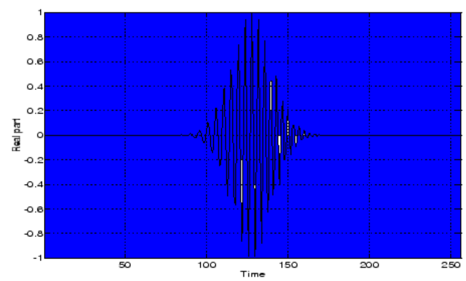


Fig.(5) : Mono-component non-stationary signal with a linear frequency modulation and a gaussian amplitude modulation

What about multi-component non-stationary signals ?

The notion of instantaneous frequency implicitly assumes that, at each time instant, there exists only a single frequency component. A dual restriction applies to the group delay : the implicit assumption is that a given frequency is concentrated around a single time instant. Thus, if these assumptions are no longer valid, which is the case for most of the multi-component signals, the result obtained using the instantaneous frequency or the group delay is meaningless.

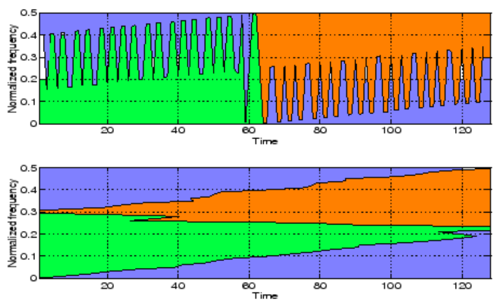


Fig.(6): Estimation of the instantaneous frequency (first plot) and group-delay (second plot) of a multi-component signal

So these one-dimensional representations, instantaneous frequency and group delay, are not sufficient to represent all the non-stationary signals. A further step has to be made towards two-dimensional mixed representations, jointly in time and in frequency. Even if no gain of information can be expected since it is all contained in the time or in the frequency representation, we can obtain a better structuring of this information, and an improvement in the intelligibility of the representation we presented a first class of time-frequency distributions of non-stationary signals. These distributions decompose the signal on a basis of elementary signals (the atoms) which have to be well localized in time and in frequency. Two well known examples of such decompositions are the short-time Fourier transform and the wavelet transform.

Seismic deconvolution by atomic decomposition: A parametric approach with sparseness constraints an alternative approach to the blind seismic deconvolution problem is presented that aims for two goals namely recovering the location and relative strength of seismic reflectors, possibly with super-localization, as well as obtaining detailed parametric characterizations for the reflectors. We hope to accomplish these goals by decomposing seismic data into a redundant dictionary of parameterized waveforms designed to closely match the properties of reflection events associated with sedimentary records. In particular, our method allows for highly intermittent non-Gaussian records yielding a reflectivity that can no longer be described by a stationary random process or by a spike train. Instead, we propose a reflector parameterization that not only recovers the reflector's location and relative strength but which also captures reflector attributes such as its local scaling, sharpness and instantaneous phase-delay. The first set of parameters delineates the stratigraphy whereas the second provides information on the lithology. As a consequence of the redundant parameterization, finding the matching waveforms from the dictionary involves the solution of an ill-posed problem. Two complementary sparseness-imposing methods Matching and Basis Pursuit are compared for our dictionary and applied to seismic data.

Wavelet transform-based spectral decomposition Once one accepts the notion that a seismogram can be represented as a superposition of wavelets, it follows immediately that the frequency spectrum of that seismogram must be a superposition of the frequency spectra of the wavelets. Thus, once a seismogram has been decomposed into constituent wavelets, a time-versus-frequency analysis (spectral decomposition) can readily be constructed by weighted superposition of wavelet spectra as a function of record time. Notably, such an approach to time-frequency analysis requires no windowing and no use of the Fourier transform if an appropriate wavelet dictionary (set of wavelets) is utilized. Consequently, the method has excellent time resolution and eliminates "Gibbs phenomena" and other undesirable effects of windowing such as spectral notches caused by multiple seismic reflection events occurring within the analysis window. We refer to our wavelet transform-based spectral decomposition technique as Enhanced Spectral Processing (ESP) in order to call attention to the fact that processing applications of the method go well beyond hydrocarbon indication.

Conclusion , Discussions & Perspectives:

Wavelets are powerful signal processing tools that have found applications in a broad spectrum of scientific and applied engineering problems. Some of the current engineering applications include image processing, communication, data storage and compression as well as information extraction for pattern recognition and diagnostics. Concepts and theories of wavelets provide a unified framework for a number of technologies developed independently for various signal processing applications including filter banks, multi-resolution and subspace analysis.

A general framework for soft-shrinkage with applications to blind deconvolution and wavelet denoising is being developed to overcome which causes problems if one wants to extract edges .In many practical applications one has to extract information out of measured data . Additionally, the noisy data usually belongs to a larger function space than the exact data, which makes it sometimes difficult to extract the information searched for; e.g. an image with piecewise continuous. A common way to overcome these difficulties is to remove the noise by some data denoising techniques and/or to compute an approximation of the exact data that belongs to the proper (smaller) function space. Initial analysis of the receiver arrays on wavelet estimation based on the Extinction Theorem : Knowing the source signature (amplitude, phase and radiation pattern) is an important prerequisite for new demultiple, imaging and inversion methods.

By using the Extinction Theorem to derive an algorithm for finding the source wavelet using only measurements of pressure along the cable/streamer

Application of multi-wavelet seismic trace decomposition and reconstruction to seismic data interpretation and reservoir characterization

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