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Finite difference modeling of P-SV wave propagation in 2D elastic media with PML boundary conditions

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Summary

In the present work, I have modeled the P-SV wave propagation in 2D homogeneous and isotropic fullspace and solid half-space using second-order accurate velocity-stress finite difference method (Virieux, 1986). The perfectly matched layer (PML) absorbing boundary conditions (Chew and Liu, 1996) are applied at edges of the computational domain in order to minimize the edge reflections. Stress image method (Graves, 1996) is used to represent the planar freesurface boundary of the half-space. Numerical examples are presented which show excellent absorption of edge reflections with PML boundary conditions.

Introduction

Finite difference methods are widely used for modeling wave propagation in elastic media. But the finiteness of the computational domain due to memory limitations creates artificial boundaries resulting in artifacts in the form of edge reflections. This often results in masking of the true events. There are absorbing boundary conditions based on paraxial approximations (Clayton and Engquist, 1977) which are computationally efficient, however they fail when the incidence angle is large. Moreover the edge reflections originating due to interaction of surface waves with artificial edges are poorly suppressed by the most of the absorbing boundary conditions. On the other hand, the perfectly matched layer (PML) boundary conditions are found to be very efficient in absorption of the edge reflections especially in absorbing the Rayleigh waves at the free surface.

Theory

I have applied the velocity-stress finite difference method (Virieux, 1986) to P-SV wave equation. This method uses the basic elastodynamic equations in terms of velocity and stress which represents a firstorder hyperbolic equation. Let us consider a 2-D medium with a horizontal axis x and vertical axis z pointing downward. The medium is assumed linearly elastic and isotropic. Then the equations are given as

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial t_{xx}}{\partial x} + \frac{\partial t_{xz}}{\partial z} \qquad (1a)$$

$$\rho \frac{\partial^2 w}{\partial t^2} = \frac{\partial t_{xz}}{\partial x} + \frac{\partial t_{zz}}{\partial z} \qquad (1b)$$

$$t_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} + \lambda \frac{\partial w}{\partial z} \qquad (2a)$$

$$t_{zz} = (\lambda + 2\mu)\frac{\partial w}{\partial z} + \lambda \frac{\partial u}{\partial x} \qquad (2b)$$

$$t_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \qquad (2c)$$

Here and are the horizontal and vertical components of displacement, (t_{xx}, t_{zz}, t_{xz}) is the stress tensor, ρ (x,z) is the density, λ (x,z) and μ (x,z) are Lame coefficients. This system is transformed into the following first-order hyperbolic system

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$$\frac{\partial v_x}{\partial t} = b\left(\frac{\partial t_{xx}}{\partial x} + \frac{\partial t_{xz}}{\partial z}\right) \quad (3a)$$

$$\frac{\partial v_z}{\partial t} = b\left(\frac{\partial t_{xz}}{\partial x} + \frac{\partial t_{zz}}{\partial z}\right) \quad (3b)$$

$$\frac{\partial t_{xx}}{\partial t} = (\lambda + 2\mu)\frac{\partial v_x}{\partial x} + \lambda\frac{\partial v_z}{\partial z} \quad (4a)$$

$$\frac{\partial t_{zz}}{\partial t} = (\lambda + 2\mu)\frac{\partial v_z}{\partial z} \quad (4b)$$

$$\frac{\partial t_{xz}}{\partial t} = \mu\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right) \quad (4c)$$

Here (v_x, v_z) is the velocity vector, b(x,z) is lightness or buoyancy which is the reciprocal of density.

The discretization of (3) and (4) leads to a unique staggered grid as shown in Fig. 1.

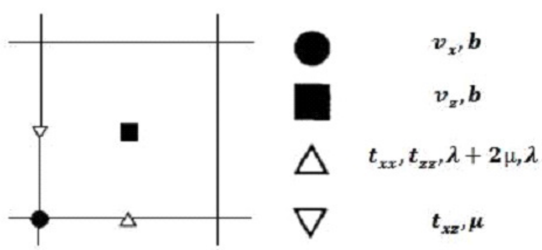

Figure 1: Discretization of medium on a staggered grid. White symbols are for stresses and Lame parameters at time $k\Delta t$. Black symbols are for velocities and buoyancy at time $(k+\frac{1}{2})\Delta t$.

In order to formulate the equations for perfectly matched layer, we need to write the equations of elastodynamics in a stretched coordinate system. This is obtained by defining the nebula operator for the stretched coordinates as

$$\nabla_e \equiv \hat{x}\frac{1}{e_x}\partial_x + \hat{y}\frac{1}{e_y}\partial_y + \hat{z}\frac{1}{e_z}\partial_z \quad (5)$$

Here, e_x, e_y and e_z are the stretching variables. Chew and Liu (1996) showed that in order to have a perfectly matched interface, the stretching variable in the direction normal to that interface need not be the same on the two sides of the interface. To understand it better, let us assume a 2D medium whose *x*-axis is along horizontal direction and *z*-axis is positive downward. Suppose a perfectly matched layer is orthogonal to *x*-axis, then $e_{1z} = e_{2z} = e_{1x} = 1$ and e_{2x} should take a complex value in order to exponentially attenuate a plane wave incident at the perfectly matched interface from the interior of the computational domain.

Let us take $e_{2x} = 1 - \frac{i\Omega_x}{\omega}$ where Ω_x is a real scalar function of the distance *x* from the interface and ω is the angular frequency. The variables are chosen to be frequency dependent since the Fourier transform of a real-valued function will always have *i*ω appearing together. For continuity reasons, Ω_x should be zero on the perfectly matched interface. Since we have taken the interface to be perpendicular to the *x* -axis, therefore $\Omega_z = 0$

I have used the second-order finite difference scheme for the PML as proposed by Festa and Nielsen (2003).

Numerical modeling and results

A point source directed vertically downward is used. The source time function is a Gaussian pulse given by

$$f(t) = e^{-\alpha(t-t_0)^2} \quad (6)$$

In order to show the effect of PML boundary conditions in case of body wave absorption, I have chosen a full-space whose physical property parameters are listed below:

Lame Parameters		Density
$\lambda\ Nm^{-2}$	$\mu\ Nm^{-2}$	$\rho\ Kgm^{-3}$
$3.0 * 10^{10}$	$3.0 * 10^{10}$	2500

The synthetic seismograms obtained at a lateral point in case of the full space with PML absorbing boundary conditions (ABC) and with rigid boundaries are shown in Fig. 2.

I have also modeled wave propagation in a solid halfspace in contact with air such that the planar free surface is horizontal and aligned with the numerical grid. In case of staggered grid, I have modeled the free surface using stress-imaging method (Graves, 1996). The physical property parameters of the half-space model are listed below:

Lame Parameters		Density
$\lambda\ Nm^{-2}$	$\mu\ Nm^{-2}$	$\rho\ Kgm^{-3}$
$3.0 * 10^{10}$	$3.0 * 10^{10}$	2500

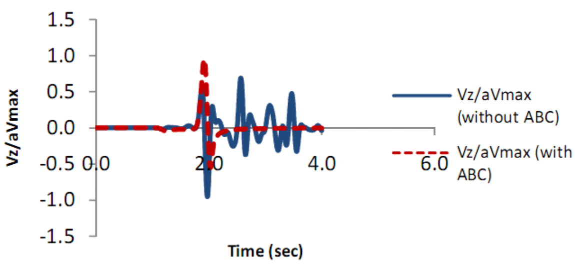

Figure 2: Vertical component seismograms obtained using a staggered grid for a point at lateral distance of 6000 m from source in an infinite medium. Edge reflections can be seen in the seismogram obtained without application of the absorbing boundary condition. The application of perfectly matched layer has completely suppressed these edge reflections.

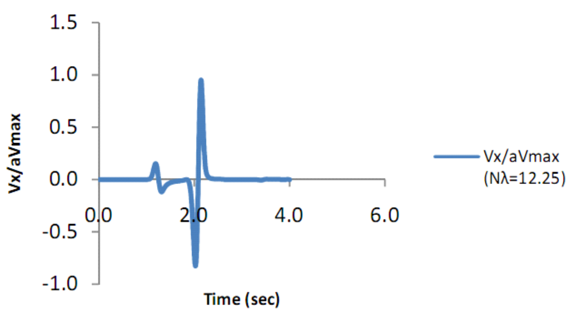

Figure 3: Horizontal component seismogram computed on a staggered grid with N_{λ} = 12.25 using stress imaging at a point shifted by half grid spacing along horizontal and vertical axes from the point at a lateral distance of 6000 m from source in a solid half-space.

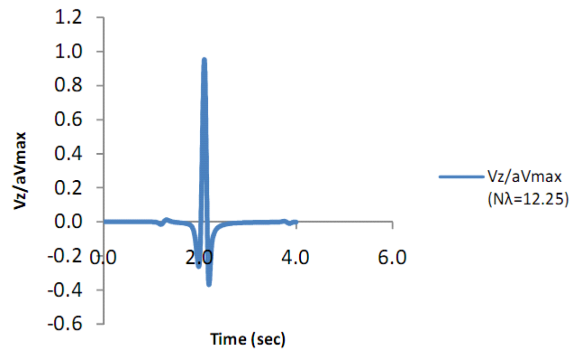

Figure 4: Vertical component seismogram computed on a staggered grid with N_{λ} = 12.25 using stress image method at a point whose lateral distance is 6000 m from the source in a solid half-space.

Conclusions

The perfectly matched layer (PML) boundary conditions are very efficient in absorption of edge reflections. PML boundary conditions provide excellent absorption of body waves as well as Rayleigh waves as shown in Fig. 2 and 3. In both the numerical examples, I have taken 10 grid points thick PML layer. Even for wavelengths much larger than the PML thickness, the absorption is very efficient.

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