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Quantification of Error Statistics in 2D Thermal Models: Application to Sedimentary Basins

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Summary

The Cambay basin is one of the major onshore oil producing sedimentary basins of India, and is approximately located between 21^0N to $24^0 30'$N and $71^0 30'$E to $73^0 45'$E in the western margin of Gujarat state. Quantification of accurate subsurface temperature field is of vital importance for the better understanding of crustal/lithospheric evolution and temperature controlled geological processes such as maturation of hydrocarbons, mineralization, hydrothermal circulation which are of economic importance. The thermal structure is influenced by its geothermal parameters such as thermal conductivity, radiogenic heat sources and initial and boundary conditions. The mathematical equations governing the behavior of the earth are constructed by combining conservation laws and constitutive relationships and are solved analytically as well as numerically using the initial and associated boundary conditions. In this study the two dimensional heat conduction equations are solved in a stochastic framework incorporating randomness in thermal conductivity both numerically and analytically. The thermal conductivity is a very important parameter for understanding the present and past thermal regimes of sedimentary basins. The bulk thermal conductivity of sedimentary rocks depends mainly on three factors such as their mineralogical and fluid composition, their temperature and their structure. As these numerical values are not known with certainty, we have considered the thermal conductivity to be a random parameter. The analytical solution to the problem is obtained using the Adomian method of decomposition. The two dimensional plot of the mean temperature-depth distribution shows that the mean temperature field is increasing with depth as expected. The plot of standard deviation, which is a measure of uncertainty in the system behavior, shows that it also increases with depth. Further with an increase in errors in the input, an increase in the errors in the system behavior is observed. The result shows that the Curie isotherm, which is the isotherm of approximately 550^oC, lies approximately at 19 km depth and the Moho temperature is found to be approximately 900^oC with a maximum of 920^oC at Tharad and a minimum of 850^oC at Degam and the standard deviation is seen to be approximately between $60\,^oC$ to $85\,^oC$. To demonstrate the thermal field in sedimentary basins a Matlab based GUI has been developed.

Introduction

Estimation of the subsurface thermal structure of the crust is required to understand large class of geological problems. The subsurface thermal field is mostly obtained in a deterministic frame work. The thermal conductivity and radiogenic heat source are used to estimate the subsurface temperature which varies both laterally and vertically in complex ways. In such cases it is not possible to infer with certainty the subsurface temperature in a deterministic framework. Therefore it is essential to solve the problem in a stochastic framework to quantify the effects of uncertainties in the controlling parameters on the temperature field. Some studies on the effects of random heat sources and conductive parameters on the temperature distribution have been carried out by Vasseur et al (1985), Vasseur and Singh (1986) and Nielson (1987).

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Quantification of any physical phenomena is done by a suitable mathematical formulation of a simple algebraic equation, an integral equation or a differential equation, relating to various parameters of interest. This mathematical model is essential to extract the fine properties of the physical phenomena and to study the characteristics of the process, which include steady state and the transient response. Mathematical formulation of any physical process involves various complex equations, which can be solved using different analytical and or numerical techniques to get the desired solutions which can be analyzed in terms of the physical phenomena. Differential equation itself contains no information related to any specific problem such as geometry of the media, nature of the conditions at boundary, initial condition etc. Differential equations are solved using two approaches, analytical and numerical methods. Most commonly used analytical methods are: method of separation of variables, integral transform method, integral balance method and Adomian method. Numerical methods such as the FEM (Finite Element Method), FDM (Finite Difference Method) can be used which can provide approximate but acceptable solutions. Analytically the heat equation has been solved using the Adomian approach where the solution is built using a series expansion method. Adomian's method of decomposition (Adomian 1994) has now been generalized as a general analytic procedure to solve deterministic or stochastic, linear or nonlinear equations. It has been shown to be systematic, robust, and sometimes capable of handling large variances in the controlling parameters. In a recent study using this new approach the stochastic heat conduction equation was solved by Srivastava and Singh (1999), incorporating uncertainties in the thermal conductivity, where the solution for the temperature field was obtained using a series expansion method. The thermal conductivity was considered to be a random parameter with a known Gaussian colored noise correlation structure. Later, Srivastava (2005) extended the study to obtain the analytical expressions for mean heat flow and its variance.

Theory

We begin by considering the linear two-dimensional heat conduction equation

$$\frac{\partial}{\partial x}(K(x,y)\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(K(x,y)\frac{\partial T}{\partial y}) = -A \qquad 0 \le x \le l_x \, and \quad 0 \le y \le l_y \tag{1}$$

where $T(x,y)$ is the temperature (^oC), $K(x,y)$ is the thermal conductivity $(W/m\,^\circ C)$ which can be represented as a sum of a mean part and a random part, A is the radiogenic hear source ($\mu W/m^3$) and l_x and l_y are the length and width of the sedimentary basin (m).

A usual representation for the spatial variability in the thermal conductivity assumes is

$$K(x,y) = \overline{K} + K^{'}(x,y) \tag{2}$$

where $\overline{K}$ is the mean and $K^{'}(x,y)$ is the fluctuation part.

Common representations for the fluctuating part in the thermal conductivity is assumed to be random with a Gaussian correlation structure

$$< K^{'}(x,y) > \; = 0$$

$$< K^{'}(x_1,y_1)\, K^{'}(x_2,y_2) > \; = \sigma_K^2 e^{-\rho d^2} \tag{3}$$

where < > denotes the expectation operator, σ_K^2 is the variance in the thermal conductivity, ρ is the correlation decay parameter($1/m^2$), or $(1/\rho)$ is the correlation length scale (m^2), and $d^2 = d_x^2 + d_y^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$ is the square of the distance between the coordinates

(x_1, y_1) and (x_2, y_2) (m^2) . Equation (1) becomes

$$\frac{\partial}{\partial x}((\overline{K} + K'(x,y))\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}((\overline{K} + K'(x,y))\frac{\partial T}{\partial y}) = -A \qquad 0 \le x \le l_x and \quad 0 \le y \le l_y \tag{4}$$

Associated boundary conditions are

$$T(0, y) = f_1(y)$$

$$T(l_x, y) = f_2(y)$$

$$T(x, 0) = f_3(x) \tag{5}$$

$$T(x, l_y) = f_4(x)$$

In one of the boundaries i.e. at the base of the model the flux condition has been considered. The solution to the problem is obtained using the procedure of Srivastava and Singh (1999) with two terms in the decomposition series. This is not a limitation of the method, but a limitation of the information available. The inclusion of additional terms in the solution would require the availability of higher-order moments in the thermal conductivity. However, it is known that when decomposition series converge, they do so very rapidly and only a few terms in the series are required for an accurate solution.

Examples

To demonstrate the methodology proposed we have considered the Cambay basin region which is one of the major onshore oil producing sedimentary basins of India .The basin runs in the form of a graben in approximately NNW-SSE direction up to $21^0\ 45'$N latitude and thereafter turns in the NNE – SSW direction towards the Gulf of Cambay. In the north, the basin joins Kutch rift basin and to its south lies the Narmada rift basin. West of the Cambay basin is the Saurashtra Peninsula covered by the Deccan traps. The Cambay basin developed sequentially from north to south during northward motion of the Indian plate after the breakup of the Gondwanaland. To model the crustal thermal structure the profile between Tharad to Degam portion has been used as shown in figure1.

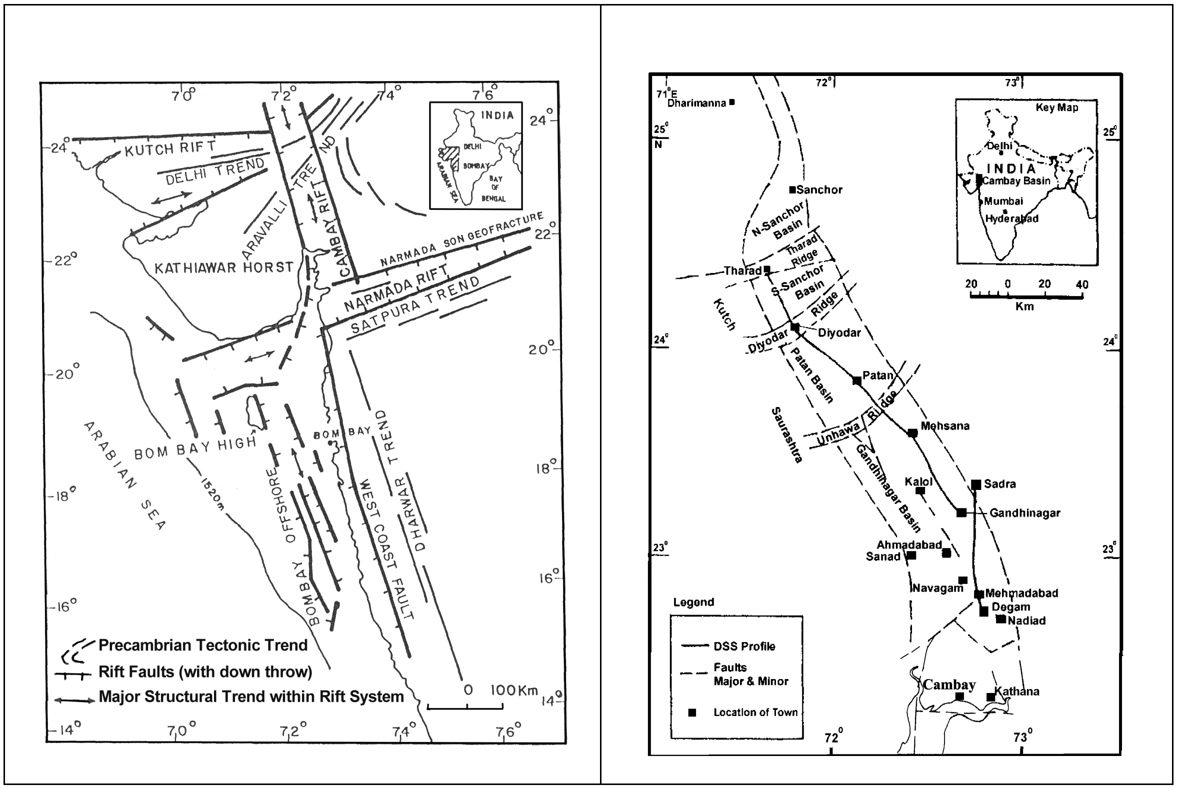

Fig 1: Tectonic and Location map of Cambay basin and a DSS profile from Tharad to Degam in the north Cambay basin

The surface heat flow values measured at Mehsana, Kalol, Sanad, Navagam, Kathana and Cambay in the northern part of the Cambay basin indicate that the northern Cambay basin is characterized by high heat flow values from 75 – 93 mW/m^2 with an average value of 83 mW/m^2 as compared to the normal heat flow value of approximately 46 mW/m^2 for stable continental areas. The heat production is ranging from 2.1 to 2.7 ($\mu W/m^3$) and the mean thermal conductivity is taken to be $K(x,y)$ around 3.0 ($W/m\,^\circ C$) and the coefficient of variability in thermal conductivity is considered between 0.2 to 0.4. The result shows that the Curie isotherm, which is the isotherm of approximately 550°C, is around 20km and the temperature at Moho is about 900°C with a maximum of 920°C at Tharad and a minimum of 850°C at Degam and the standard deviation is approximately between 60 $^\circ$C to 85°C.

Conclusions

The analytical solution to the subsurface temperature field and its associated error bounds have been obtained by solving the steady state heat conduction equation incorporating Gaussian uncertainties in the thermal conductivity using the Adomian method of decomposition.

First the solution to the temperature field is built using a series expansion method. Closed form analytical expressions for mean and variance in the temperature depth distribution have been obtained and an automatic formulation has been developed in mat lab for computing and plotting the thermal structure. Using a sequence of mat lab m files a simple graphical user interface (GUI) viewer has been developed which allows us to give the controlling thermal parameters on the screen directly and it displays the subsurface thermal structure along with its error bounds. The software developed can be used to quantify the thermal structure for any given region. The thermal conductivity is assumed to be a realization of a Gaussian random process and the mean behavior of the temperature fields along with its error bounds is obtained. The temperature and its errors are seen to increase with depth. The temperature along the profile do not vary significantly due to homogeneity along the horizontal direction.

The temperatures and its associated error statistics can be used for a better evaluation of the thermal state of sedimentary basins for hydrocarbon maturation.

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