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Use of Prior and Hyper-prior in Seismic Inversion for Reservoir Characterization

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Summary

Seismic data are devoid of high and low-frequency informations due to the band limitation of the wavelet. This results in non-unique estimates of earth model parameters in the missing frequency bands. Here we review some popular techniques and advocate some new strategies to navigate the 'null-space' so that realistic model estimates can be made from observations. In the inversion of NMO corrected gathers, missing low frequency band (typically below 5 Hz or so) must come from independent information such as interpolated well logs. A typical band-limited inversion would result in oscillations like Gibbs' phenomenon near the layer boundaries. Imposing sparse spike constraint using Lp norm of the model parameters or a Tikhonov regularization generally reduces this. We have developed an alternate albeit a more efficient scheme based on Gaussian hyper-prior that easily imposes blockiness in the derived model. Finally, missing high frequencies are typically incorporated using a stochastic inversion in which a priori models are drawn from a statistical distribution derived from the well logs. To this end, we propose the use of a fractional Gaussian distribution that offers a more realistic statistic of earth model parameters.

Introduction

The ultimate goal of seismic inversion and analysis is to produce a pseudo-log at every surface location so that a detailed 3D map of a reservoir can be constructed (e.g Sen 2006). Seismic data are almost always inadequate, insufficient and inconsistent. Therefore, this can only be achieved by meaningful integration of seismic, well log and other ancillary data such as core and petro-physics. In other words, seismic inversion can be viewed as a 'physics based interpolator' that creates rock property profiles at every possible location.

The task of crating rock property profile from seismic data is highly challenging in that only a limited amount of information is contained in the seismic data and the problem is non-linear and mathematically ill-posed. Realizing the fact that no mathematical trick can fill in the missing information exactly, attempts are made to better pose the problem using mathematical regularization techniques such as Tikhonov regularization (first or second order smoothing) and model constraints such as spikiness in reflectivity or blockiness in acoustic impedance. Deterministic algorithms that incorporate such constraints are commonly used in seismic inversion packages.

In this paper we demonstrate a new approach to ascertain blockiness in acoustic impedance (AI) and shear impedance (SI) based on a Bayesian hyper-prior. Further, to incorporate high frequency variations that match the well log frequency spectrum, we devise a non-linear optimization approach in which starting models are drawn from a fractional Gaussian distribution estimated from well data. We demonstrate effectiveness of these approaches with application to synthetic and field seismic data

Method

To address the fundamental problem, we show a typical wavelet and its corresponding amplitude spectrum in Figure 1. Notice the missing low and high frequency information in the spectrum. This problem is fairly well known. Missing low frequency band can only come from *a priori* information such as the well logs. However, we need to choose the pass-band carefully. Inadequate low-frequency model will result in incorrect estimates of

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absolute AI values. An example is shown in figure 2 where AI inversion was done was using three high cut frequencies, namely, 10Hz, 5 Hz and 0.5 Hz. The results are compared against the well log at the same location. It is obvious that while top and middle panels show adequate reproduction of the well logs, the lower panel shows that the seismic derived AI fails to match the well logs.

One additional problem with missing low-frequency band is that it causes oscillations at the layer boundaries and this problem is particularly severe in modeling thick layers. This is addressed by incorporating smoothing or sparse spike constraint, which was applied in the results shown in Figure 2. This can be better done using Gaussian Hyper-priors described below.

Gaussian Hyper-prior

A standard Bayesian formulation to solve nonlinear inverse problem can be stated as

$$P(m \mid d^{obs}) = \frac{P(d^{obs} | m) P(m)}{P(d^{obs})}$$

where P(m) is the prior model distribution and P(dobs|m) the likelihood function. In the Bayesian hyper-prior formulation proposed by Calvetti and Somersalo (2007), the prior distribution can be further expressed by $P(m) = P(m|\theta)P(\theta)$

where theta is the prior distribution parameter. Thus we have

$$P(m \mid d^{obs}) = \frac{P(d^{obs} | m) P(m|\theta) P(\theta)}{P(d^{obs})}$$

The choice of the distribution is critical to the Bayesian approach. Assuming Gaussian error model, Calvetti and Somersalo (2007) choose the distribution of the likelihood and the conditional distribution to be Gaussian where as the distribution of the hyper parameters are chosen to be Cauchy like distribution with long tail. Choosing a Cauchy distribution for the prior parameters gives the ability to produce sparse distribution for parameters. This is justified since the jumps in the model are expected to occur only at certain locations. The Gaussian hyperpriors are employed in the seismic inversion problem by Routh et al (2008). In this approach a CGLS (conjugate gradient least squares) was used for optimization.

Stochastic inversion using fractional Gaussian distribution

In general, all inversion algorithms rely on good starting models to produce realistic earth models. A new method based on a fractional Gaussian distribution derived from the statistical parameters of available well-logs to generate realistic initial models was proposed by Srivastava and Sen (2009). The method uses fractal theory for the generation of these models. A global optimization method called 'very fast simulated annealing' (VFSA) is used in finding the minimum of an objective function that minimizes data misfit and honors the statistics derived from well logs. The proposed stochastic inversion method addresses missing frequencies due to the band-limitation of the wavelet by combining the low and high frequency variation from well logs with seismic data. While the models derived by a deterministic inversion are devoid of high frequency variations present in the well log, those derived by the new stochastic inversion reveal high frequency variations that are consistent with seismic and well log data

Examples

Figure 3 shows an example of AI inversion from seismic data using Gaussian hyper-priors. The results are compared against those using standard regularization techniques such as an L_2 model norm, total variation (TV – a proxy for an L_1 norm) and wavelet basis norms (Routh et al 2008). It is fairly clear that the new Gaussian hyper-prior works the best in estimating details of the variations in the well log.

In figure 4, we show an example of pre-stack seismic inversion that uses angle gathers as input data (Srivastava and Sen 2009). A fractional Gaussian distribution characterized by a mean, a variance and a Hurst coefficient is derived from the well log. Starting models are drawn from the fractal based distribution and synthetic seismograms are generated. A VFSA (Sen and Stoffa 1995) based optimization method is used to update the model parameters until we obtain an adequate fit of the seismic data. A comparison of the results obtained from a deterministic and stochastic inversion is shown in Figure 4. The superiority of the stochastic method in tracking high frequencies is clearly demonstrated.

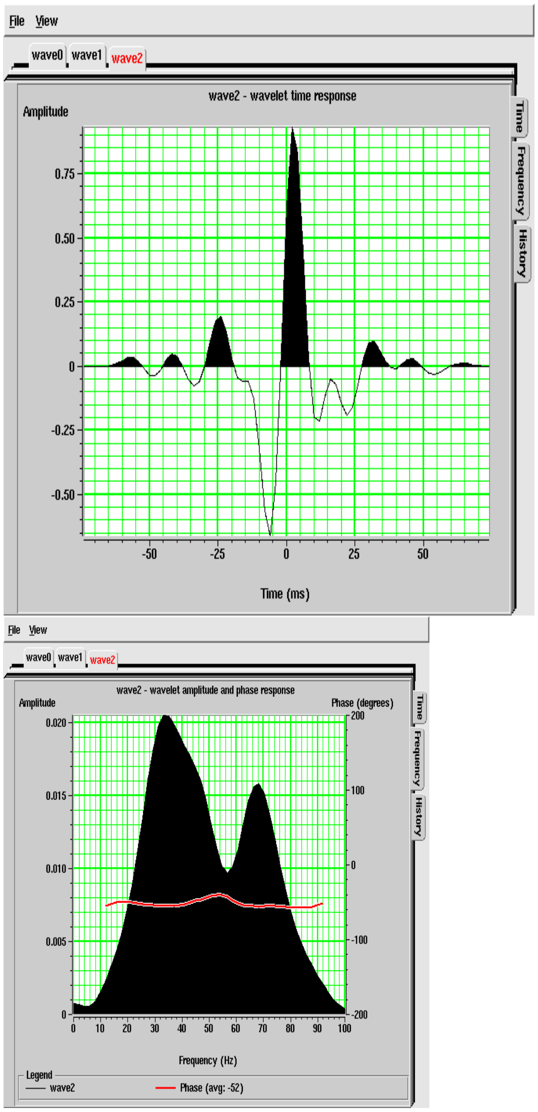

Figure 1: A typical wavelet (top panel) derived from a seismic trace and its corresponding amplitude spectrum (bottom panel). Note the missing low- and high-frequency bands in the spectrum.

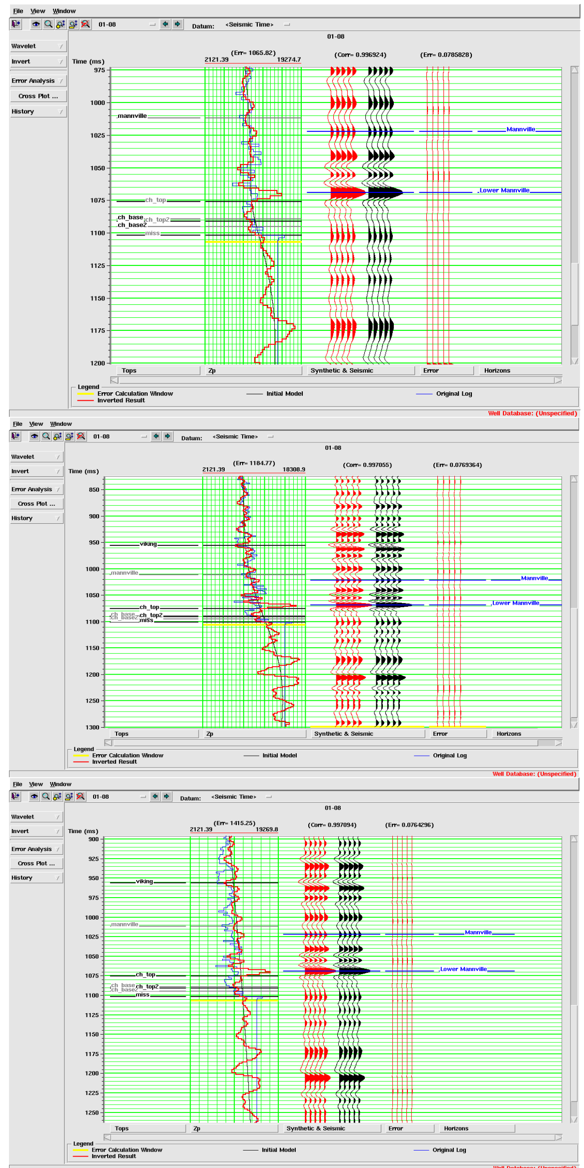

Figure 2: Seismic inversion results using three different low frequency starting models, top: high cut 10Hz; middle: high cut 3Hz and bottom: 0.5Hz. Note that although the datafit is equally good for all three statrting models, the estimates of absolute impedance values are grossly in error for the lowest panel.

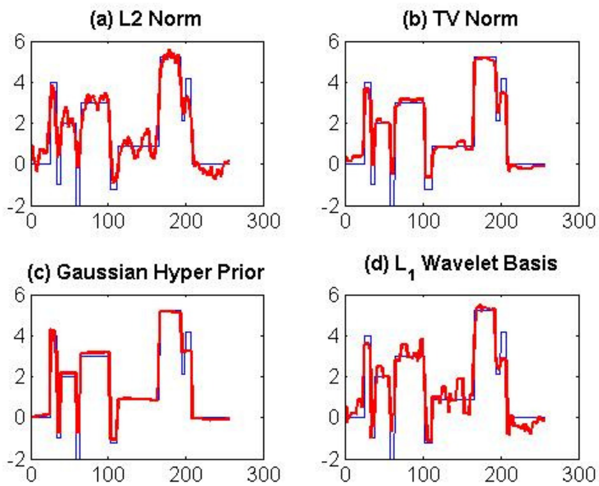

Figure 3: Inversion of seismic data for AI using different regularization techniques. The true model is shown in black while the inverted models are shown in red. Note the superior performance of Gaussian hyper-prior compared to other well known regularization method (from Routh et al 2008).

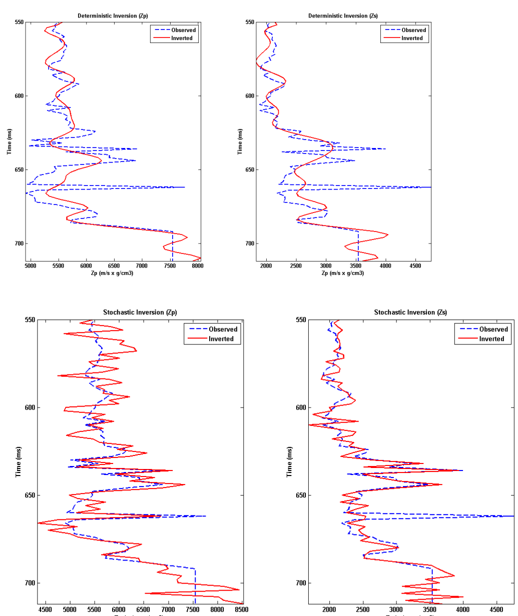

Figure 4: Pre-stack inversion results: AI and SI estimates from a deterministic (top) and fractal based stochastic (bottom) inversion (from Srivastava and Sen 2009).

Conclusions

In this paper we demonstrate two approaches to incorporating realistic a priori information and model constraints. The Gaussian hyper-prior approach incorporates blockiness and thus supplies some of the missing high frequency informations. Such an approach, when used in a deterministic framework will perhaps not be successful in bringing out thin layers (below seismic travel time resolution). On the other hand, the fractal based stochastic inversion supplies missing low and high frequencies as starting model statistically, which are verified and accepted, based on seismic data fit. Note that the entire missing high frequency band in the wavelet does not reside in the null space. Thin layers, if any, do affect amplitude responses and can thus be modeled statistically.

Our future work will include a combination of Gaussian hyper-prior and fractal based stochastic approach. This approach will likely lessen the dependence on statistic derived from well logs and will be better suited in areas with few wells. One other modification we propose is to replace VFSA with CGLS for faster convergence of the stochastic inversion.

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