



P-406

## Estimation of Seismic Q Using a Non-Linear (Gauss-Newton) Regression

Parul Pandit <sup>\*a</sup>, Dinesh Kumar <sup>b</sup>, T. R. Murali Mohan <sup>a</sup>, Kunal Niyogi <sup>a</sup>, S.K. Das <sup>a</sup>

<sup>a</sup> GEOPIC, Oil & Natural Gas Corporation Ltd, Dehradun, India

<sup>b</sup> Department of Geophysics, Kurukshetra University, Kurukshetra, India

### Summary

It has been observed that the amplitudes of seismic waves decay faster than predicted by geometrical spreading of the wave fronts. The attenuation of seismic waves due to inelastic nature of medium is described by quality factor,  $Q$ . The reliable estimates of  $Q$  are required to design an inverse  $Q$  filter which accounts for the absorption and dispersion effects from the recorded seismic data. This is useful for meaningful amplitude inversion and reservoir characterization. The classical log-log spectral ratio method may give biased  $Q$  estimation due to log transformation of data. A non-linear (Gauss-Newton) technique has been presented here to estimate seismic  $Q$ . The validity of the technique has been demonstrated by estimating  $Q$  from the reflection data. We found that the  $Q$  values estimated using Gauss-Newton method are lower than those of estimated using classical spectral ratio method. The estimated frequency independent  $Q$  values (21 – 32) indicate the presence of water saturated sand in the region (Jannsen et al, 1985). The method can be applied to more number of traces of same section to infer more information about the region.

### Introduction

The true amplitude information of the seismic data is required for the amplitude inversion and reservoir characterization. The amplitudes of seismic waves are influenced by the source strength as well as intervening medium between the source and receiver. The amplitudes of seismic waves generally decrease with depth. The attenuation properties of the media are inherent elements governing the amplitude of seismic waves at various frequency levels. The overall attenuation is composed of several factors that include geometrical spreading, scattering due to inhomogeneities in the media and anelasticity. The amplitudes of the seismic waves decay faster than predicted by geometrical spreading of wave fronts. The geometrical spreading controls the amplitudes of seismic waves in the homogeneous and purely elastic earth. As the real earth is not perfectly elastic the net loss of energy during seismic wave propagation is symbolized by absorption. The absorption of seismic waves is partly controlled by intrinsic physical loss mechanisms such as

internal friction and partly by inhomogeneities along the travel path that can cause scattering. Due to the non-elastic nature of the medium, a part of the energy in the wave is dissipated instead of being transferred through the medium. This type of attenuation of seismic waves, known as intrinsic/anelastic attenuation or damping, is described by a parameter called quality factor,  $Q$ . The quality factor,  $Q$ , which measures the deviation from perfect elasticity is defined as (Knopoff, 1964):

$$1/Q = - (1/2\pi) (\Delta W/W) \quad (1)$$

where  $\Delta W$  is the energy lost in one cycle and  $W$  is the total energy available in a harmonic wave. The attenuation coefficient,  $a$ , is related to  $Q$  as (Futterman, 1962) :

$$Q = 2\pi [1 - \exp(-2\alpha\lambda)] \quad (2)$$



where  $\lambda$  ( $=2\pi c/\omega$ ) is wavelength,  $c$  is speed of wave propagation and  $\omega$  is frequency.

The reliable estimates of  $Q$  are needed to design an inverse  $Q$  filter which accounts for the absorption and dispersion effects from the recorded seismic data. After applying inverse  $Q$  filter, the seismic data is expected to have better resolution useful for reliable seismic inversion. Wang (2008) has shown the effectiveness of inverse  $Q$  filtering to the seismic data.

A number of techniques for estimating the  $Q$  from the seismic data have been proposed. These include rise time method (Gladwin and Stacey, 1974; Kjartansson, 1979), analytical signal, phase modeling, wavelet modeling, spectrum modeling, frequency modeling and spectral ratio methods. Tonn (1991) has investigated and compared the different methods of  $Q$  estimation.

The spectral ratio is widely used and discussed in detail by Bath (1974). In this method the amplitude spectra of wavelets at two different levels are estimated. The usual way to estimate  $Q$  using spectral ratio method is to linearize the equation of spectral ratio by taking logarithms of both sides and then to use the least square criterion. The results of least square inversion on log transformed data may be biased (e.g. Menke, 1984). The Gauss-Newton technique can be applied to estimate  $Q$  if error in the data is normally distributed (e.g. Lee and Stewart, 1981). We presented here a non-linear technique (Gauss-Newton) to estimate seismic  $Q$  from the spectral ratio without linearizing the equation. In this technique the model parameters are estimated iteratively by expanding the relationship around the model parameters using a Taylor series expansion. The validity of the technique has been demonstrated by seismic  $Q$  from the reflection data.

### Method

The amplitude spectrum,  $A_1(\omega)$ , of a wavelet reflected at depth  $Z_1$  can be written as (Jannsen et al, 1985):

$$A_1(\omega) = A_0(\omega)G(Z_1)R_1 \exp(-2\alpha_1 Z_1) \quad (3)$$

$A_0(\omega)$  is the amplitude spectrum of the incident wavelet at  $Z = 0$ ,  $G(Z_1)$  accounts for the geometrical spreading and  $R_1$

is the reflection coefficient. The amplitude spectrum of a reflection from the depth  $Z_2$  ( $> Z_1$ ) is given by:

$$A_2(\omega) = A_0(\omega)G(Z_2)(1-R_1^2)R_2 \exp(-2\alpha_1 Z_1) \exp(-2\alpha_2 (Z_2 - Z_1)) \quad (4)$$

where  $(1-R_1^2)$  accounts for the two way transmission across the interface. The spectral ratio (SR) of two spectra can be written as (dividing equation 4 by equation 3) :

$$SR(\omega) = C_1 \exp(-2\alpha_2 (Z_2 - Z_1)) \quad (5)$$

where  $C_1 = [G(Z_2)/G(Z_1)][(1-R_1^2)R_2/R_1]$  and phase velocity  $c$  are assumed to be independent of frequency in spectral ratio (Bath, 1974). Taking natural log of equation (5) gives:

$$\ln SR(\omega) = \ln(C_1) - \alpha_2 \Delta T c \quad (6)$$

where  $\Delta T$  is the time difference between the two reflections. Substituting the value of  $c$  from equation (2) we get

$$\ln SR(\omega) = \ln C_1 + \omega \Delta T / 4\pi \ln(1 - 2\pi/Q) \quad (7)$$

which is a equation of straight line between  $\ln SR(\omega)$  and  $\omega$  whose slope (f) gives the  $Q$  as

$$Q = 2\pi / [1 - \exp(4\pi f \Delta T)] \quad (8)$$

This is the widely applied spectral ratio method to estimate  $Q$ . Now we describe the Gauss-Newton method to estimate the  $Q$  from the spectral ratios. In this technique we do not linearize the equation by taking natural log (as in case of classical spectral ratio method) but use it directly i.e. equation (7) without log can be written as:

$$SR(\omega) = C_1 (1 - 2\pi/Q)^{\omega \Delta T / 4\pi} \quad (9)$$

In terms of data and model parameters, the above equation can be written as:



$$d^{obs} = f(m) \quad (10)$$

where  $d^{obs}$  is the observed data [= SR ( $\omega$ )] and  $f(m)$  is function of model parameters (i.e.  $C_1$  and  $Q$  in this case). In Gauss Newton technique,  $f(m)$  is expanded around the initial model ( $m^0$ ) using Taylor's series. If  $y$  represents the difference between the observed data and the synthetic data estimated from the initial model, then inverse problem of equation (10) can be written as (Meju, 1994):

$$E = y - Ax \quad (11)$$

where  $A$  is the matrix of the partial derivative of  $f(m)$  with respect to each of model parameters  $m_j$ . The vector  $x$  contains the unknown corrections to be determined and applied to the initial model  $m^0$  so as to minimize  $E$ . Thus one has to search for the corrections or perturbations to the initial model. The solution vector of parameter perturbations of equation (11) is given by (Meju, 1994):

$$x = (A^T A)^{-1} A^T y \quad (12)$$

where  $A^T$  is transpose of  $A$ . The perturbation is applied to the initial model  $m^0$  to get the better solution  $m^1$ . The procedure is repeated using  $m^1$  as the new starting model. The iterative formula is given by:

$$m^{k+1} = m^k + (A^T A)^{-1} A^T y \quad (13)$$

### Application

The non linear Gauss Newton method described above has been applied to the 2D seismic reflection offshore data with record length of 12 seconds. Figure 1 shows location of the traces (in a box) on the section used in the present analysis.

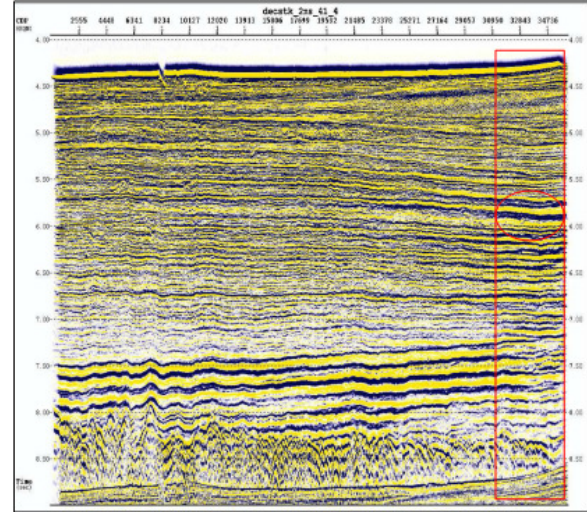


Figure 1: Location of traces on the seismic section used in this study

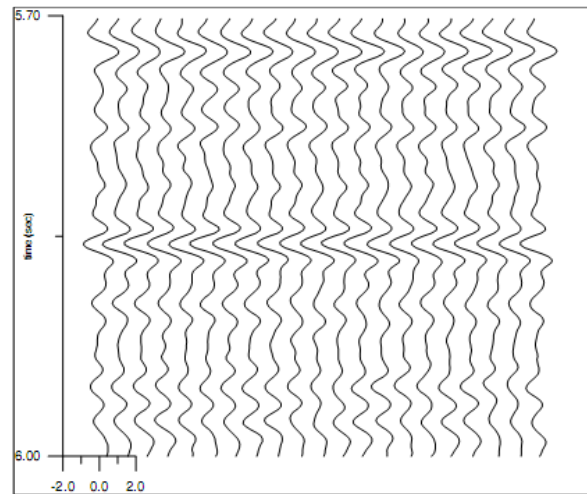


Figure 2: A set of seismic traces used in the present study

A set of individual traces are displayed in the Figure 2. These traces show the two significant reflected signals at time 5.71 sec and 5.84 sec.



## Estimation of Seismic Q Using a Non-Linear (Gauss-Newton) Regression

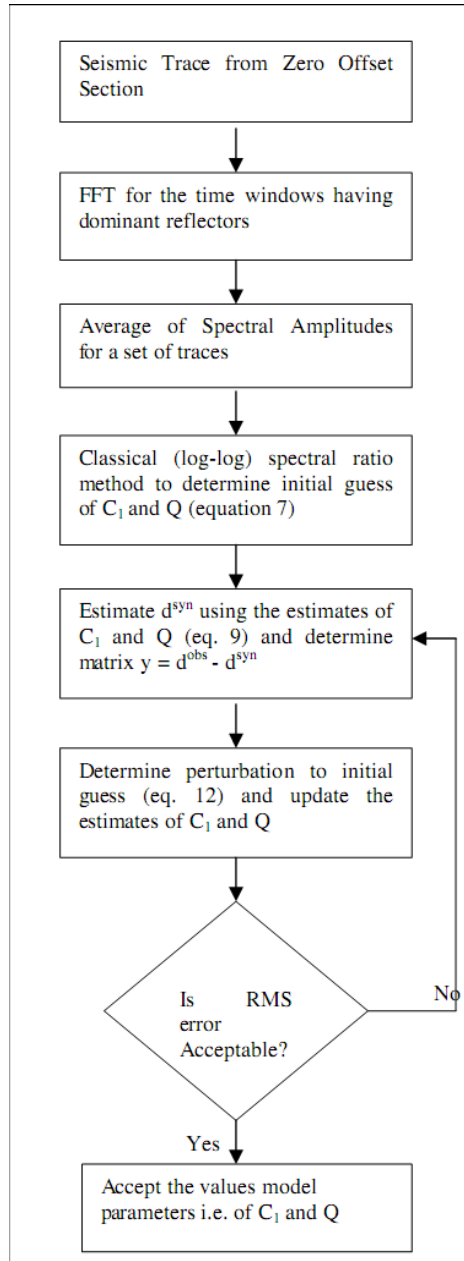


Figure 3: A flow chart to estimate Q using Gauss Newton method.

Figure 3 shows the flowchart to estimate the values of model parameters  $C_1$  and  $Q$  using Gauss Newton method. The data set has been divided into three sets and  $Q$  values are estimated for each set. Figure 4a shows the Fourier spectra of two reflected events and Figure 4b displays the log of spectral ratio versus frequency along with the least square fitted line for one set of seismic traces.

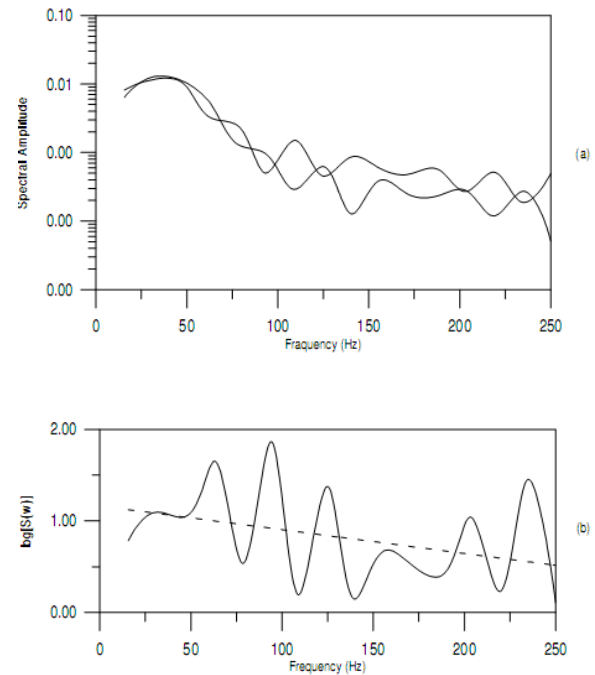


Figure 4 (a) Fourier Spectra of two reflected events (b) Spectral ratio versus frequency along with the least square fitted line.

The initial values of  $Q$  estimated for three sets using classical (log-log) spectral ratio method are 22, 25 and 41 respectively. After applying the Gauss Newton method, these values modified to 21, 20 and 32 respectively. We note that the modified  $Q$  values are lower than those of estimated using classical spectral ratio technique. Similar  $Q$  values can be estimated for more set of seismic traces from the same section. The estimated  $Q$  values (21-32) are corresponding to water saturated sand at this depth level. However this can be validated using more number of traces along the same profile line. Such analysis may be done for deeper layer also. The  $Q$  values are useful for designing an



## Estimation of Seismic Q Using a Non-Linear (Gauss-Newton) Regression



inverse Q filter to remove absorption and dispersion effects from the recorded seismic data.

### Conclusion

A non-linear (Gauss Newton) method has been applied to determine the seismic Q from the reflection data. The estimated Q values are different from those of estimated using classical (log-log) spectral ratio technique. The frequency independent Q values are found to be varying in the range 21 - 32. These values indicate the presence of water saturated sand at this depth level (Jannsen et al, 1985). This may be validated using more number of traces of same profile. The estimated values are important for the designing of an inverse Q filter to improve the resolution of seismic data and thus useful for the reservoir characterization.

### References

- Bath, M., 1974, Spectral analysis in Geophysics, in Developments in Solid Earth Geophysics, Elsevier, Amsterdam
- Futterman, W.I., 1962, Dispersive body waves, Journal of Geophysical Research, 67, 5279-5291.
- Gladwin, M.T. and Stacey, F.D., 1974, Anelastic degradation of acoustic pulses in rock, Physics of the Earth and Planetary Interiors 8, 332-336.
- Jannsen, D., Voss, J. and Theilen F., 1985, Comparison of methods to determine Q in shallow marine sediments from vertical reflection seismograms, Geophysical Prospecting 33, 479-497.

Kjartansson, E., 1979 Constant Q propagation and attenuation, Journal of Geophysical Research 84, 4737-4748.

Knopoff, L., 1964, Q, Reviews of Geophysics, 2, 625-660.

Lee, W.H. and Stewart, S.W., 1981, Principles and applications of microearthquake networks, Academic Press, New York, 293p.

Meju, M.A., 1994, Geophysical data analysis: understanding inverse problem, theory and practice, SEG, 299p.

Menke, W., 1984, Geophysical Data Analysis: Discrete inverse theory, Academic Press, Florida, 269p.

Tonn, R., 1991, The determination of seismic quality factor Q from VSP data: A comparison of different computational methods, Geophysical Prospecting, 39, 1-27.

Wang, Y., 2008, Seismic inverse Q filtering, Blackwell Publishing, 234p.

### Acknowledgements

The authors place on record their sincere thanks to Director (Exploration), ONGC, for his kind permission to publish this work. The authors are thankful to Mr. V.S. Bhatnagar, Mr. Surendra Kumar, Mrs. Mamta Jain, Mr. A.V.S. Sarma and Mr R.S. Rana for their kind support and cooperation. The authors are indebted to Miss A. Kavitha for her generosity and motivation.

*The views expressed in this work are solely of the authors and do not necessarily reflect the views of ONGC.*