



## Sequence Stratigraphy by Wavelet Analysis

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### Keywords

Geophysical Well Logging, Wavelet transform, Gamma logging, Sequence stratigraphy, Milankovitch cycles

### Summary

Geophysical well logging (GWL) data is nonstationary in character. It detects cyclicity, trends, sudden changes in sedimentation and stratigraphy. Wavelet analysis is more effective for analyzing nonstationary signals in which a signal can be represented as the sum of different frequency components with different resolutions. The driving impetus behind wavelet analysis is their property of being localised in time (space) as well as scale (frequency). This provides a time-scale map of a signal, enabling the extraction of features that vary in time. This makes wavelets an ideal tool for analysing signals of a transient or non-stationary nature. Geophysical well-log (bore-hole) data represent the rock physical properties as a function of depth measured in a well. They aid in demarcating the subsurface horizons, identifying abrupt changes in physical properties of rocks and locating cyclicity in stratal succession. Since wavelet transformations can better identify the abrupt changes in cyclicity common in nature, they become important tools for sequence stratigraphy. Currently spectral decomposition methods (Continuous Wavelet Transform, Matching Pursuit Algorithm decomposition, Discrete Wavelet Transform) are used to detect hydrocarbon zones. The wavelet transform is a multi-scale operator and is well known to point out singularities in the analysed signal. The way in which the wavelet transform analyses the signal can be compared to the geoscientist's interpretative behaviour in the hydrocarbon upstream industry: both look at the signal at different scales (frequency range), detect breaks "major events" and heterogeneity and characterise trends. In order to measure multi-fractility of any signal the Holder Exponents and Singularity Spectrum attributes are computed. Mathematical transformations are applied to signals to obtain a further information from that signal that is not readily available in the raw signal. Wavelet analysis generates useful information from well-log responses. It is a useful tool for automated cyclostratigraphy. Wavelet analysis is a multiresolution framework and, thus, it is well suited for upscaling rock and flow properties in a multiscale heterogeneous reservoir. The compact support property of the wavelet transform assures efficient computation. Choice of regularity provides a flexible way to control the smoothness of the resulting upscaling properties. Because the upscaled property images obtained from wavelet transform capture the characters of the original property fields, the predicted performance from

upscaling property fields matches well with that from the original fine-scale property fields. Morlet wavelet is best suited to gamma ray logging signal to identify cyclic geological information like sequence stratigraphy. Detrended fluctuation analysis of well logging data provides precise sequence stratigraphy. MATLAB Wavelet Toolbox is employed for GWL data analysis.

### Introduction

Scaling paradigm proves fruitful in image processing, scale relativity and fractal time-space. The central notion behind scaling laws is that of self-similarity. Wavelet analysis is a multiresolution framework and, thus, it is well suited for upscaling rock and flow properties in a multiscale heterogeneous reservoir. The compact support property of the wavelet transform assures efficient computation. Choice of regularity provides a flexible way to control the smoothness of the resulting upscaling properties. Because the upscaled property images obtained from wavelet transform capture the characters of the original property fields, the predicted performance from upscaling property fields matches well with that from the original fine-scale property fields. The driving impetus behind wavelet analysis is their property of being localised in time (space) as well as scale (frequency). This provides a time-scale map of a signal, enabling the extraction of features that vary in time. Currently spectral decomposition methods (Continuous Wavelet Transform, Matching Pursuit decomposition, Discrete Wavelet Transform) are used to detect hydrocarbon zones. Hydrocarbon reservoirs are detected at low frequencies. The wavelet transform is a multi-scale operator and is well known to point out singularities in the analysed signal. The way in which the wavelet transform analyses the signal can be compared to the geoscientist's interpretative behaviour. In the hydrocarbon upstream industry: both look at the signal at different scales (frequency range), detect breaks "major events" and heterogeneity and characterise trends. In order to measure multi-fractility of any signal the Holder Exponents & Singularity Spectrum attributes are computed. Wavelets in well-test applications have focused on denoising and data conditioning. Further work could be performed to implement wavelets as a useful tool in well-test interpretation. MATLAB Wavelet Toolbox is employed for logging data.

### Theory /Methods

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**Wavelet Analysis:** Wavelet analysis represents an advance from Fourier analysis through a windowing technique with flexible regions. WT uses a window function whose radius increases in space (reduces in frequency) while resolving the low-frequency contents of a signal. WT is an analysis tool well-suited to the study of multiscale, nonstationary processes occurring over finite spatial and temporal domains. The integral (continuous) WT of a function  $f(t)$  with respect to some local base function (wavelet)  $\psi$  is defined as

$$W(b, a) = \frac{1}{\sqrt{a}} \int f(t) \psi^* \left( \frac{t-b}{a} \right) dt, \quad a > 0$$

where  $\psi^*$  is the complex conjugate of  $\psi$ . The parameters  $b$  and  $a$  are called translation (shifting) and dilation parameters, respectively.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi \left( \frac{t-b}{a} \right) \quad a, b \in \mathbb{R}; \quad a \neq 0$$

Although  $W(b, a)$  provides space-scale analysis rather than space-frequency analysis, proper scale-to-frequency transformation allows analysis that is very close to space-frequency analysis. Reducing the scaling parameter  $a$  reduces the support of the wavelet in space and hence covers higher frequencies, and vice versa. Therefore,  $1/a$  is a measure of frequency. The parameter  $b$  indicates the location of the wavelet window along the space axis. Thus, changing  $(b, a)$  enables computation of the wavelet coefficients  $W(b, a)$  on the entire space-frequency plane. A required condition for  $\psi$  is that all wavelets must oscillate, giving them the nature of small waves and hence the name wavelets. The CWT separates out the frequency components of a signal. It is therefore important that the wavelet used gives the best resolution in frequency. The shape of the wavelet coefficients at some scale should resemble a sinusoid at the corresponding pure frequencies. The best wavelet for this purpose is the Morlet wavelet with its Gaussian modulated complex decaying exponential. The graphical representation of the wavelet coefficients for the different scales (wavelengths) as a function of depth is the scaleogram.

One of the most widely used continuous wavelets in geophysics is the complex Morlet wavelet, which consists of a plane wave modified by a Gaussian envelope, given by

$$\psi(t) = \exp(-t^2/2) \exp(2\pi j f_0 t)$$

where  $f_0$  is the fundamental frequency of the wavelet. Other continuous wavelets include the Mexican Hat and Shannon. Here, we use the Morlet wavelet because it has a cyclic component which will "resonate" with any cyclic

components in  $f(t)$ . Wavelet analysis reveals both small-scale (high frequency) and large-scale (low frequency) aspects of signals which can be overlooked by Fourier analysis.

The basic idea of wavelet transform of logging signal is that a family wavelet function  $\psi_{a,b}(t)$  is used to approximate logging signal  $f(t)$ , expand logging signal from onedimensiondepth domain to two-dimension depth-scale domain, and form a measure with relative amplitude on coordinate position of depth (or space)  $b$  and scale factor (time period or spatial scale)  $a$ . Results of continuous wavelet transform are wavelet coefficients, which are the function of scale parameter  $a$  and translation parameter  $b$ . The value of wavelet coefficient reflects the similarity between wavelet function and logging signal. It can be interpreted as covariance between logging signal function and wavelet function.

Logging signal is a function with argument being depth  $t$ . A logging signal  $f(t) \in L^2(\mathbb{R})$ , its continuous wavelet transform is defined as:

$$C_f(a, b) = \langle f, \psi_{ab} \rangle = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{+\infty} f(t) \psi \left[ \frac{t-b}{a} \right] dt$$

Where  $a$  is a scale parameter controlling sample interval of logging signal, and  $b$  is the translation parameter controlling approximating space of logging signal.

Analyzing from aspect of energy,  $C_f(a, b)$  reflects local characteristics centered around  $(a, b)$  in time-frequency domain of signal  $f(t)$ , or is interpreted as energy spectrum of function  $f(t)$  in negligible time-frequency window.

Therefore,  $|C_f(a, b)|^2$  reflects local energy of signal  $f(t)$  on  $(a, b)$  in time-frequency domain.

**Wavelets Boundary Cyclostratigraphy :** Wavelet transform is a multiresolution analysis because it analyses the signal at different frequencies with different resolutions. The low scale (compressed) wavelet investigates high frequency components of signal with good resolution in time but poor in frequency domain. The high scale (stretched) wavelet investigates low frequency components with good resolution in frequency domain but poor time resolution. This is the result of Heisenberg's uncertainty principle that applies to information theory and signal processing : the exact time and frequency of a signal can never be known simultaneously, i.e., a signal cannot be plotted as a point on the time-frequency plane. Two explanations of the origin of sedimentary cycles are changes in water depth, which are largely caused by tectonic movements, and changes in climate. These causes may overlap and an analysis of the sedimentary record may not permit a clear distinction

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between the two. Sea level changes related to orbitally driven climate cycles are generated by orbital rhythms known as Milankovitch cycles.

#### Data Analysis Steps

One-dimension continuous wavelet transform of logging signal includes five basic steps . Step 1: Collecting a wavelet function and making it align with initial point of logging signal. Step 2: Calculating wavelet coefficient  $C$ .  $C$  reflects similarity between wavelet function and collecting section of signal. The bigger the  $C$  is, the higher the similarity is. The computational solution depends on the form of wavelet function. If energy of signal and wavelet function is 1 respectively,  $C$  can be interpreted as correlation coefficient. Step 3: Making wavelet function move rightwards one unit along time axis, repeating Step 1 and Step 2 until the wavelet function covers the total signal. Step 4: Making wavelet function stretch one unit and repeating Step 1 to Step 3 . Step 5: Repeating Step 1 to Step 4 on all scales.

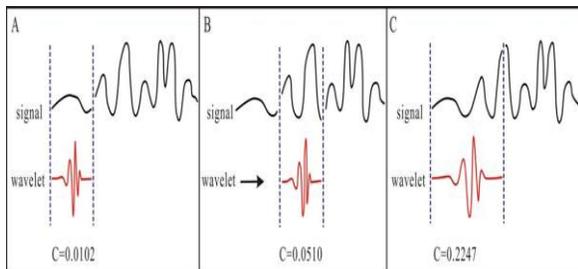


Figure 1: General step of wavelet transform of logging signal

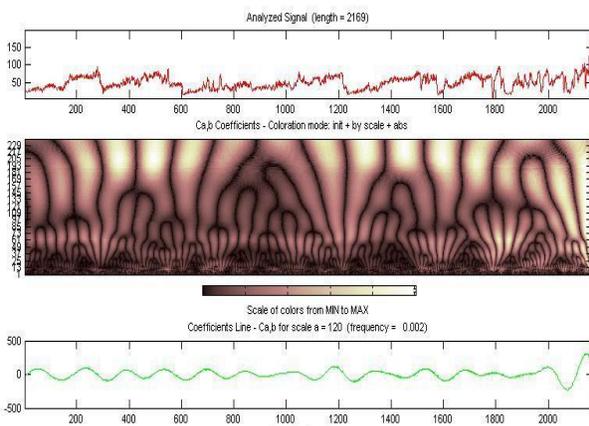


Figure 2: Wavelet analysis of Gamma logging data

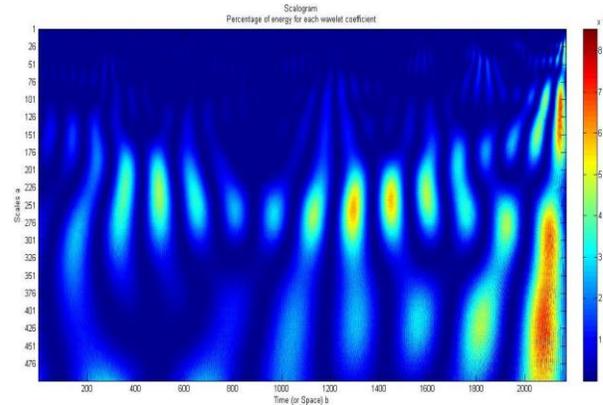


Figure 3: Scalogram

CWT of Logging Signal: Wavelet analysis represents an advance form of Fourier analysis through a windowing technique with flexible regions. WT uses a window function whose radius increases in space (reduces in frequency) while resolving the low-frequency contents of a signal. It is an analysis tool well-suited to the study of multi-scale, non-stationary processes occurring over finite spatial and temporal domains. The CWT separates out the frequency components of a signal (Fig. 4a). Well Logging Data is acquired from National Methane Hydrates R&D Program - Mount Elbert Well Log Data, Milne Point, Mount Elbert Gas Hydrate Well Site, in Arctic Northern Alaska 70.4556 N 149.4108 W, BP-DOE-DOE, USA. This is a Gamma ray (GR) logs record of spontaneous emission of gamma rays from the radioactive elements hosted in the minerals of rocks.

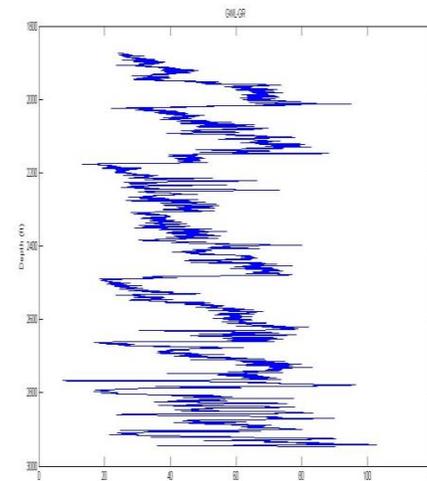


Fig. 4a: Well logging signal, Gamma log.

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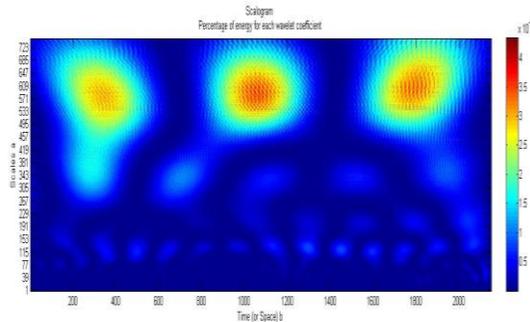


Fig.4b: Scalogram

### Results and Discussion

Logging signal includes much geology information, is the main data of identifying sedimentary cycles. By the continuous wavelet transform, the logging signal is expanded from one dimension depth domain into two-dimension depth-scale domain. In the process, time-frequency characteristics of logging signal are obtained. Based on the wavelet coefficients scalogram and oscillation characteristics of wavelet coefficients lines, different sequence boundaries can be identified easily. This method is an effective complement for traditional logging sequence stratigraphy by providing a new idea and an efficient approach for the quantitative division of sequence.

**Continuing Challenges For Wavelet Analysis :** While the preceding examples suggest that wavelet analysis may help detect cyclic patterns, limitations remain. For example, the Morlet wavelet is natural for detecting repetitive events because of its oscillating character. It is unable, however, to discriminate between fining-upwards and coarsening-upwards cycles because the Morlet wavelet is symmetrical. To discriminate between fining-upwards and coarsening-upwards cycles, we need an asymmetrical wavelet. Such wavelets exist (e.g., Daubechies and Haar), but the interpretation of the results is not as straightforward as using Morlet to detect cyclicity. Boundary detection is another area of work in progress. The greatest chances for success using the Morlet wavelet analysis appear to be when cyclic patterns are present, wavelengths change across the boundary, and multiple measurement types respond to the change in facies. Other types of boundaries, where there may be little or no cyclicity, may be better identified using another type of wavelet.

### Conclusions

Geophysical logging signal is various geophysical parameters of stratum, including electrochemical property, electrical property, acoustic property, and radioactivity, being obtained by logging instrumentation measuring subterranean formation along exploration bore hole. The logging signal has the highest resolution and the best consecutiveness and contains much geological information, including lithology, sedimentary cycle, palaeoenvironment and paleoclimate. The normal logging signal is the discrete signal sampled by equal interval in depth space. Based on the theory of signal analysis, it can be regarded as the non-steady discrete depth signal. The continuous wavelet transform is an efficient technique for detecting and quantifying the characteristics of certain types of patterns of variation in 1-dimensional data (e.g. cyclic, thinning-up and thickening-up). Identification of zones of the permeability data with characteristic scales of heterogeneity is best informed by using the continuous wavelet transform. Good localisation of high permeability contrasts in data using discrete wavelet transformations can be achieved. Wavelet analysis generates useful information from well-log responses. It is the appropriate tool for automated cyclostratigraphy. Log and core measurements, responding to different rock and fluid properties, can give different scaleogram results. The combined results from several logs is desirable to define zone boundaries or assess cyclicities. The DWT plays an important role in dividing a complicated signal into several simpler ones and analyze them separately. This concept is known as multiresolution analysis. Here, the function is decomposed at various levels of approximations and resolutions. As a result, a well-log can be decomposed into a low frequency approximation and several medium-to-high frequency details. Each individual approximation or detail can be analyzed separately, depending on the application. The simplest orthogonal wavelet is the Haar wavelet, which is based on a box function. Other widely used orthogonal wavelets are the Daubechies wavelets with different orders that are compactly supported. There are also semiorthogonal wavelets such as splines wavelets, and biorthogonal wavelets. Detrended fluctuation analysis of well logging data is innovative research of wavelet analysis for precise sequence stratigraphy. Recognizing important cyclic patterns in rocks may suggest their causes and allow prediction of reservoir quality and extent.



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