Reflectivity Enhancement using Basis Pursuit Linear Programming

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Keywords

Sparse Layer Inversion (SLI), Basis Pursuit Inversion (BPI), Linear Programming (LP)

Summary

SLI has become a prominent tool to extract high resolution structural reflectivity and enhance the hidden thin layers. Generally BPI penalizes larger coefficients more than the smaller coefficients in the reflectivity series. This boosts the low amplitude coefficients and hence ghost layers are visible which are not actually present in the original seismic.

Introduction

SLI uses an assumption of superposition that a single trace is a simple addition of multiple layer responses. If a thin layer is subdued by side-lobe interference of two adjacent layers, the subdued layer can be reconstructed using reflectivity estimation. The effect of low frequency wavelet is eliminated by SLI methods. BPI uses a sparsity constraint to reconstruct a seismic trace using limited number of layer responses.

Widess [1] defined the theoretical resolution limit to be as the 1/8th of the wavelength (λ/8). In practice, the tuning thickness can be found at 1/4th of the wavelength (λ/4). Chopra et al. [2] have used spectral inversion methods to resolve layers below the tuning thickness. Puryear and Castagna [3] have further increased the resolution to 1/16th of the wavelength by spectral decomposition. Nguyen and Castagna [4] used matching pursuit decomposition (MPD) to decompose a Seismic trace into individual layer responses. Matching Pursuit exhibits lateral discontinuity due to interference between similar elements of the wedge dictionary. Zhang and Castagna [5] proposed basis pursuit inversion for seismic data which has an additional feature of sparsity norm which provides greater lateral stability despite of the intereference between adjacent atoms of the dictionary.

The universal differentiability of l2 norm is useful for error minimization. The only disadvantage of l2 norm is that it puts a minimal weight on small residuals and substantial weight on large residuals. It enhances the suppressed noise content in the signal during the inversion process. l1-norm approximation puts less weight on large residuals [6], compared to l2-norm approximation. So the usage of l1-norm for thin layer detection can be useful since it will penalize all the coefficients equally and the thin layer components will not be boosted.

Theory

The forward model for Seismic trace S (t) can be written as follows:

$$S(t) = W(t) * R(t) + N(t) \qquad (1)$$

Which is just a simple convolution of reflectivity series R (t) and stationary wavelet W (t). The presence of noise is denoted by N (t).

It can be explained as a source wave W (t) traveling through the Seismic layers and undergoing refraction as well as a reflection at the boundaries of such layers due to the difference in layer impedances which results in the variation of wave velocity. The conventional basis pursuit method is used to present the input signal with a minimum number of atoms from the dictionary of responses.

Basis Pursuit is an optimization routine which minimizes the l1-norm and hence induces sparsity in the solution vector. It minimizes the following objective function:

$$\min[||d - Gm||_2 + \lambda||m||_1] \qquad (2)$$

Where d is the Seismic trace column vector, m is the coefficient vector, G is the kernel matrix

It uses l2-norm for error minimization and l1-norm for the penalization of associated dictionary atoms. Direct application of this method to Seismic data puts a minimal weight on the subdued thin layer coefficients as well as the noise content in the data. It degrades the efficiency of inversion due to the occurrence of ghost layers in the inverted seismic. To avoid this kind of problem and put more weight in the penalization of thin layers, we have modified the conventional basis pursuit by converting the least square solution into a least absolute solution. This converts the combination of l(21)-norm into a linear programming problem. [9]

Formulation

To modify the basis pursuit problem, the least-squares solution has been converted into the least absolute solution. This provides less constraint on the major coefficients than minor reflection coefficients. Hence the objective function can be rewritten as

$$\min[||d - Gm||_1 + \lambda||m||_1] \tag{3}$$

This can be substituted in the following form.

$$\min(y + \lambda z) \tag{4}$$

The constraint matrix in the form of Ax ≤ b can be given as

$$\begin{bmatrix} G & -1 & 0 \\ -G & -1 & 0 \\ 1 & 0 & -1 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} m \\ y \\ z \end{bmatrix} \le \begin{bmatrix} d \\ -d \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

The modified algorithm is applied to 3D Seismic data in the western part of India. Statistical wavelet is extracted from the Seismic Volume. The extracted zero-phase wavelet is shown in Fig. 2(a). Seismic well tie between the original Seismic and a particular well is shown in Fig. 2(c). Using this wavelet, wedge dictionary is constructed for a minimum thickness of 2 ms and a maximum thickness of 10 ms. Inversion is carried out using sparsity constraint λ = 0.0001 with both the algorithms.

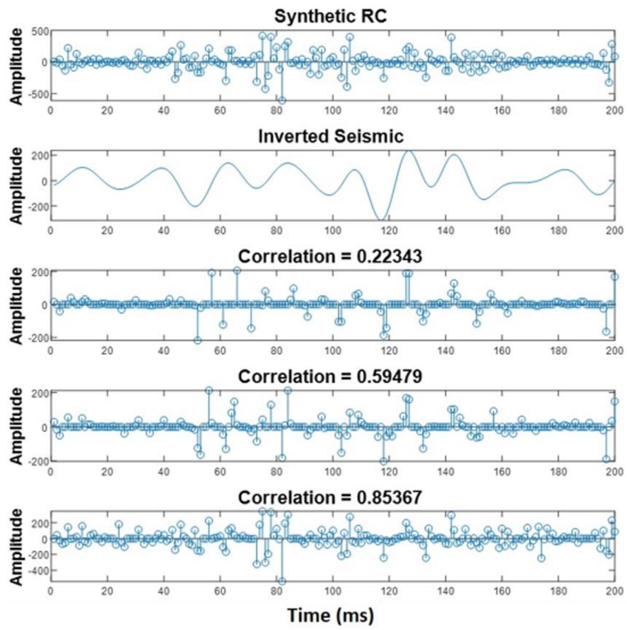

Figure 1: Inversion results using conventional basis pursuit

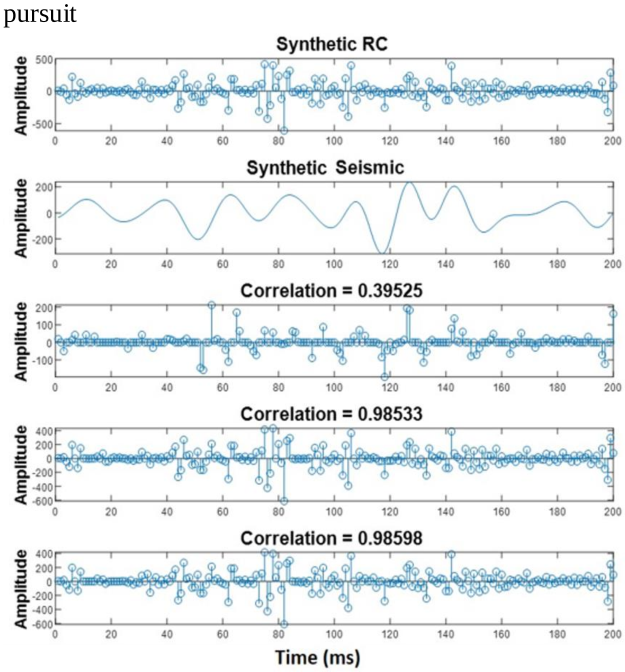

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Figure 2: Inversion results using modified linear basis pursuit inversion

Data Tests

For synthetic data testing, we have extracted reflectivity from a certain well log and convolved a 30Hz central frequency ricker wavelet to impose destructive side-lobe interference. Both the conventional and modified linear basis pursuit have been applied to the trace at different sparsity constraints. The correlation values for conventional basis pursuit inversion is shown in Figure 1. For the same sparsity constraint, the modified method shows better correlation as shown in Figure. 2.

We have applied the modified algorithm to offshore seismic data in the western part of India. In the original seismic data, most of the thin layers are subdued because of the lower bandwidth of seismic data. Since a lower frequency wavelet is convolved with the original reflectivity series, side-lobe interference has caused cancellation of most of the layer responses. The reflectivity series obtained from both the algorithms are shown in Figure.3(b) and Figure.3(c). It is evident that the number of layer occurrences have increased in the reflectivity obtained from conventional basis pursuit. This is due to the lesser constrained inversion of insignificant coefficients. The modified basis pursuit has been able to extract the reflectivity more accurately.

No starting models are required for the inversion process. Hence the algorithm is only dependent on the sparsity constraint and the tuning thickness of the dictionary. Furthermore, this algorithm solves the least absolute solution which inherently promotes sparsity. However, the efficiency of inversion heavily depends on the dictionary matrix and therefore on the exact and accurate estimation of wavelet.

Conclusions

Since reflectivity is an important attribute of seismic volume which shows the exact geometry of the adjacent layer interference, reflectivity estimation from seismic data can be really useful in fault detection and facies prediction. Hence the objective should be to delineate thin layers with appropriate regularization. The modified basis pursuit algorithm can be really useful for detecting thin layers without boosting noise components

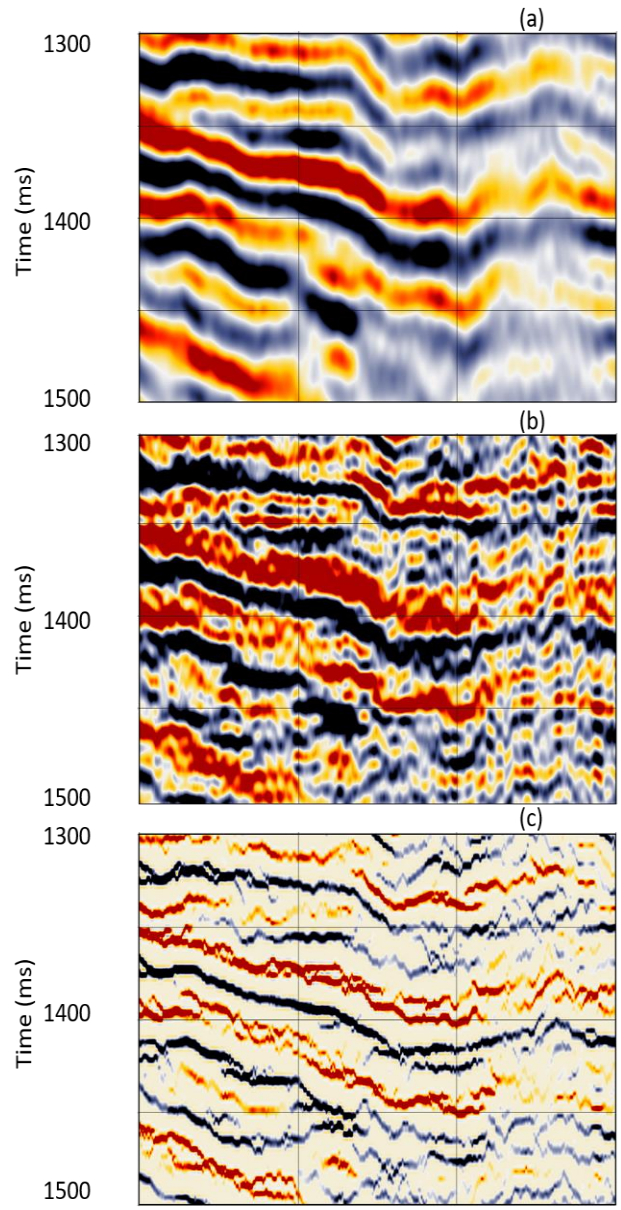

Figure 3: (a) Original Seismic Data (b) Reflectivity series obtained from conventional basis pursuit (c) Reflectivity series obtained from modified basis pursuit linear programming

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