Concepts of Map Projection-a Review

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Abstract

The real earth is converted to a mathematical surface, called spheroid/ellipsoid.A spheroid with a defined origin and orientation w.r.t the real earth becomes datum. Map projection is an attempt to portray curved surface of the earth onto a flat surface with minimal distortion. Geodesy and Cartography deal in these aspects of science which are mathematic rich. However, one may get an understanding of the process by imagining literal process through analogy. The paper presents basic concepts of the subject which are relevant to the need of the industry.

Introduction

Geodesy is a branch of mathematics for accurately measuring the Earth's size, shape, orientation, mass distribution and the variations of these with time. **Cartography** is the science and artof graphically representing a geographical area, usually on a flat surface such as a map. Every map user should have a basic understanding of map projections no matter how much computers seem to have automated the process. In oil industry, we regularly deal with location of wells, geophysical surveys, platform positions etc.

World over, every region follows its own **Coordinate reference system** (**CRS**) depending upon its geographical location. If, enough care is not taken to document the CRS system of the coordinates, locations will be mispositioned. Thus, maps and technical reports must furnish the CRS without any ambiguity. Any assumption made on the CRS, based on hearsay, would compromise on the quality. However, **Global Positioning System(GPS),** introduced subsequently, is capable of rectifying the problem to a large extent.

By the late '90, it was felt to rationalize the system by adopting a uniform CRS (WGS-84 as datum and Universal Transverse Mercator as projection system) across the globe. Following this, all the existing well coordinates are to be converted to the new CRS. It may give rise to problem of data mismatch if proper conversion parameters are not used. Therefore, it is required to have proper and thorough documentation of the CRS parameters.

Geodesy and cartography are mathematics rich disciplines and are beyond the scope of this paper. However, one may get an understanding of the process by imagining literal process through analogy. In this paper, the author has tried to represent certain basic concepts of the subject, relevant to the need of the industry.

Coordinate Reference System defines, with the help of coordinates, how a two-dimensionally projected map is related to real locations on the earth. The process is carried out in the following two successive steps.

1. In the first step, a mathematical model(spheroid/ellipsoid) is conceived and aligned with the real earth to form a Datum (3D globe). A point on the earth is located by latitude and longitude **(Geographic Coordinate Systems).**
2. In the second step we convert the datum surface (3D globe) to a flat sheet, called a map(2D surface) where we locate a point on the globe by X/Easting and Y/Northing, called **Projected Coordinate System (2D Cartesian coordinates**)

This is further discussed in detail.

Earth Surface, Geoid &Spheroid

The real surface (Fig.1A) of the Earth is never a perfect sphere. It is flattened at poles and bulges at equator. **Geoid** is a three dimensional surface of equal gravitational potential which coincides with mean sea level in the ocean and its extension into the continents through imaginary canals. Geoid is also undulated due to local variation of gravity which reflects lateral and vertical variations in the mass inside the Earth. A mathematically modelled surface is required for defining coordinate system on the earth. **Ellipsoid** is an approximate model of the shape of the earth, which best fits the Geoid. As its major and minor axes do not vary greatly it is often called a **Spheroid.**

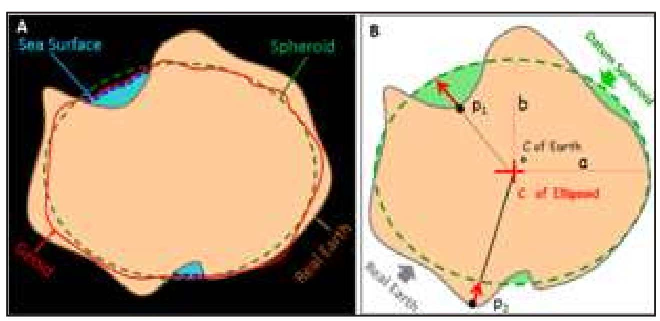


Fig.1: Representation of earth surface

A spheroid has two basic parameters,1. Semi-major axis, “a” 2. Semi minor axis, “b”/ flattening “f=(a-b)/a”. Some of the spheroid used in defining the shape and size of the earth are given in Table-1.

Once the spheroid is defined, question arises: How to fit the spheroid, the mathematical model, to the real earth?

Table 1: Selected reference ellipsoids/spheroids

Selected Reference Ellipsoids

Ellipse	Semi-Major Axis (meters)	1/Flattening
Airy 1830	6377563.396	299.3249646
Bessel 1841	6377397.155	299.1528128
Clarke 1866	6378206.4	294.9786982
Everest 1830	6377276.345	300.8017
Fischer 1968	6378150.0	298.3
G R S 1980	6378137.0	298.257222101
Hough 1956	6378270.0	297.0
International	6378388.0	297.0
Krassovsky 1940	6378245.0	298.3
South American 1969	6378160.0	298.25
WGS 72	6378135.0	298.26
WGS 84	6378137.0	298.257223563

Peter H. Dana 9/1/94

Datum

Often the term datum is confused with the term spheroid. **Spheroid is aligned with Earth to become Datum** (**Globe,** Fig.1B). Datum defines the **origin** and **orientation** of the spheroid with respect to the earth and subsequently defines **geographic coordinate systems** (lat, long and height from the spheroid). Subsequently, all the surface points are migrated onto the datum surface axially along radius of the spheroid.

The direction of the minor axis of the spheroid is aligned parallel to the mean spin axis of the earth.The position of the center of the spheroid differs from the center of mass of the earth (**Geodetic Datum**).The zero of longitude is conventionally the Greenwich Meridian.

Because there are different ways to fit the mathematical model (spheroid) to the surface of the Earth, there are many datums. Each country has developed its own datum (reference frame) independently which best fits the region.

Thus, adatum has got one & only one spheroid. But same spheroid may be aligned with the real earth differently to form different datums. Centre of spheroid may differ from center of earth. In the Fig.2, spheroid-1 (red) is aligned with North America to form datum-1(red) and the same spheroid-1 is aligned differently to form the datum-2to suite the South America. In the third picture, a different spheroie-3 (green) is aligned with Europe to form a different datum-3.

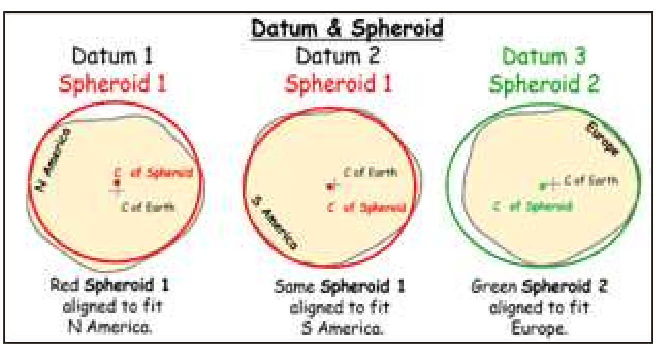


Fig. 2: Concepts of spheroid and datum

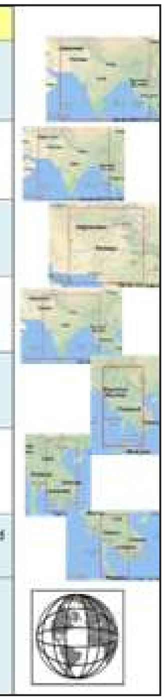
For practical purpose, the datum is technically defined by three parameters.

1. Ellipsoid - define size and shape (a and f)
2. Origin (Initial Point/fundamental point) - define geodetic coordinate value (φ-latitude λ-longitude &N-geoid/ellipsoid separation) for a chosen initial point.
3. Orientation – given by the distance and azimuth of another point w.r.t initial point (origin). Once the datum is defined, points on the earth Surface are projected radially along center of the spheroid on to datum surface to form **globe**.

Several of such datums have been customized for different parts of the world. Some of these, related to the Indian subcontinent, are given in Table-2.

Table 2: Datums used in Indian sub-continent

Datum	Fundamental Point	Spheroid/ Major Axis	
Kalianpur 1880	Kalianpur Lat: 24°07'11.260"N, lon: 77°39'17.570"E	Everest 1830 Modified a: 6377304.063 m f: 300.8017	Asia-Bangladesh, India, Myanmar, Pakistan
Kalianpur 1937	Kalianpur. Lat: 24° 07'11.260"N, lon: 77°39'17.570"E	Everest 1830 (1937 Adj) a:6377276.345 m, f: 300.8017	Asia-Bangladesh
Kalianpur 1962	Kalianpur. Lat: 24° 07'11.260"N, lon: 77°39'17.570"E	Everest 1830 (1962 Def)	Pakistan onshore
Kalianpur 1975	Kalianpur. Lat: 24° 07'11.260"N, lon: 77°39'17.570"E	Everest 1830 (1975 Def) a:6377299.151 m, f: 300.8017255	India onshore
Indian 1954	Extension of Kalianpur 1937 over Myanmar and Thailnd	Everest 1830 (1937 Adj) a:6377276.345 m, f: 300.8017	Asia- Myanmar and Thailand onshore
Indian 1960	DMA extension over Indo-China of the Indian 1954 network adjusted to better fit local Geoid	Everest 1830 (1937 Adj) a: 6377276.345 m, f: 300.8017	Asia-Cambodia and Vietnam-onshore
Indian 1975	Fundamental point: Khau Sakaerang	Everest 1830 (1937 Adj) a: 6377276.345 m, f: 300.8017	Thailand-onshore and offshore Gulf of Thailand
WGS-84 (Geocentric)	Defined through a consistent set of station coordinates. Last modified on 20/1/2002 [WGS 84 (G1150)].	WGS 84 a: 6378137 metre f: 298.257223563 Revision Date: 06/02/1995	Designed for whole Earth



Everest Spheroid & Kalianpur Datum

The **Indian Datum** is the preferred datum for India andseveral adjacent countries in Southeast Asia. It is computed on the **Everest Spheroid** with its origin at **Kalianpur**, in central India. It is largely the result of the work of Sir George Everest, Surveyor General in India from 1830 to 1843. However, there are many different versions of Indian datum that are in use to suit different parts of the India and adjoining region.

Global Datum, WGS 84

Thus choice of datum depends on the accuracy required for the purpose of the survey. Localized datum can give a more accurate representation of the area of coverage than the global datum. However, as the benefits of a global system outweigh the greater accuracy, the global **World Geodetic System 1984 (WGS84,** Fig.3A)datumis becoming increasingly adopted. Historically, the goal of geodesy has been to obtain one common datum for coordinates. WGS84

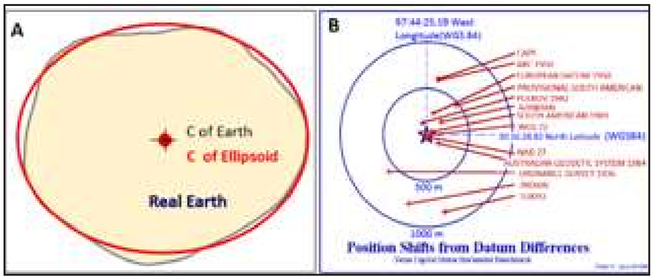


Fig. 3: Global Datum, WGS 84 geocentric Datum.

(previously WGS60 WGS66 WGS72) is defined and maintained by the United States National Geospatial-Intelligence Agency (NGA) and is used by the Global Positioning System (GPS).

The main characteristics of WGS1984 datum are:1. Ituses Spheroid WGS84 2. Both Centre of Earth and center of Spheroid coincides (**Geocentric**) 3. Globally fits the whole the Earth.

Shift of origin of different datum with respect to WGS84 global datum is given in the Fig.3B.

Latitude & Longitude of a point

It is interesting to know that a point fixed on globe could have different Lat/Long depending upon its datum (Fig.4).A wrong information on datum could shifts the location of a point by hundreds of meters. Thus Lat/Long data should always be accompanied with information of its datum.

	WGS-84	WGS-72	Kalianpur
Lat	19 00 19.320	19 00 19.162	19 00 24.028
Long	72 05 43.330	72 05 43.330	72 05 34.027

Fig. 4: Latitude & Longitude of a point and datum

Map Projection (Cartography)

Projection is a process which transfers the curved surface of the modeled earth (datum/globe), along with the latitude and longitude lines (graticules) drawn on it, on to a flat piece of paper - called a **map**. A location on the earth is represented by latitude "ϕ" and longitude "λ"which is a polar coordinate system. But, for all computational purposes, a 2D Cartesian system (**Projected Coordinate System**) is always preferred and easy to visualize (Fig.5).

No matter how sophisticated the projection process is, the original surface features can never be perfectly converted to a flat map. Projecting a curved surface onto a flat surface is always associated with some form of distortion in map properties. Different map projections retain the following geometrical properties.

1. Area (equivalent):A map which portrays shape (area preserved) accurately is called a conformal map. A conformal map tends to get quite distorted, especially towards the poles.
2. Shape (Conformal): Conformal projections preserve the relative local angles (shape preserved) about every point on the map, so that meridians intersect parallels at 90^{o} No map can be both equal-area and conformal
3. Direction(Azimuthal): Azimuthal **projection** projections show correctly the directions from all points on the map to the center. However area & shape get distorted.
4. Scale: No map projection shows scale correctly throughout the entire map; equidistant projections show true scale between two points or along every meridian.

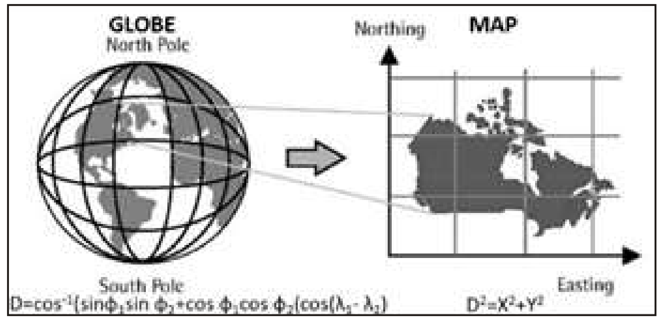


Fig. 5: Map Projection

An important part of the cartographic process is understanding distortion and choosing the best combination of projections for the job type. Some projections minimize distortions in some of these properties at the expense of maximizing errors in others. Some projection, however, moderately distort all these properties.

Broadly, the **methodology** of projections are classified into three (Fig.6) types such as **cylindrical**, **conical** and **planar**. The globe's curved surface projected on a flat map wrapped around the globe as a cylinder, produces a cylindrical map projection. Projected on a map formed into a cone gives a conical map projection. When projected directly onto the mapping plane, it produces a planar (planar or zenithal) map projection.

Although, modern projection involves a rigorous

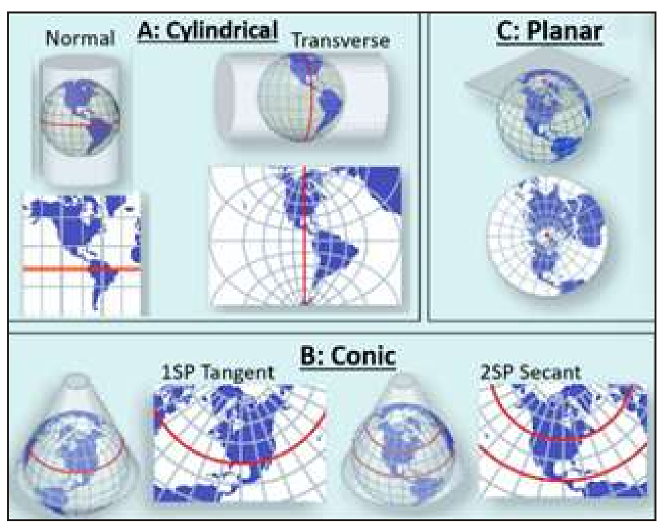


Fig. 6: Three classes of map projections.

mathematical transformation of data on a globe to respective flat surface, the process maybe understood through projection using light. If a light were to be placed in the center of the transparent globe (Fig.7), shadow of the graticules and continents would cast on the respective projected surface (cylinder, cone, and plane). If the cylinder or cone were then cut and laid flat, there would be a map. The other way to understand the process, to be discussed later, is to peel an orange appropriately, and press it forcibly on a flat surface.

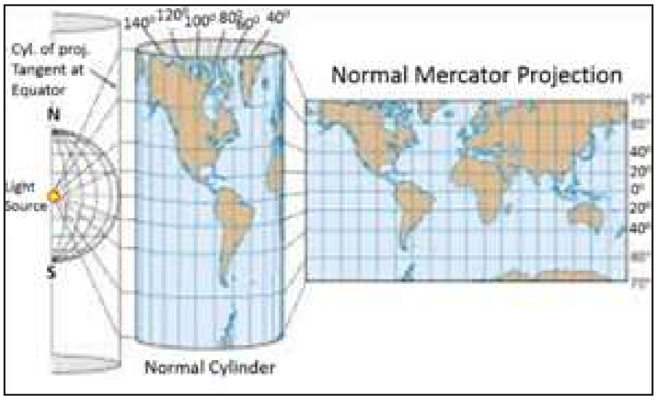


Fig. 7: Understanding map projection through light rays

In the present paper, we will be discussing cylindrical (**Universal Transverse Mercator)** projection and conical (**Lambert Conic Conformal)** projection type in detail which are relevant to oil industry in India. The planar (azimuthal) projection is used to map regions near to poles.

Universal Transverse Mercator (UTM) projection

The cylindrical projection may be parallel (normal) or perpendicular (transverse) to the spin axis of the earth (Fig.6A). We will consider the transverse (conformal)case in this paper. The Normal Cylindrical projection (equal area) is used to prepare political/thematic maps (Fig.6A & 7).

Align the transverse cylinder (Fig.8A), wrapped around the globe, in such a way that it becomes tangent to Meridian 75^0. Project a portion of the curved globe surface from 72^0 to 78^0 (3 deg either side of the meridian 75^0) onto the cylinder and then cut lengthwise (Fig.8B) and unwrap to form a flat map (Fig.8C). You will notice that except the central meridian and equator, all other graticules look curved.

A bigger peel (Fig.9), in shape of a hemisphere, will be distorted if flattened to a plane surface. However a small portion, cut as seen in the figure, could be flattened with minimal distortion which would resemble a UTM zone.

Now draw a graph (pink lines, a 2D Cartesian coordinate system, Fig.10), on the flat map for northern hemisphere, parallel to the central meridian with its origin **(Grid origin)** such that the crossing of equator and CM gets a coordinate value X=5,00,000 m, Y=0 m. This crossing is called **natural origin** and the coordinates are called **False Easting (FE) and False Northing (FN).** The false Easting is given such a large number to avoid negative values within the zone.

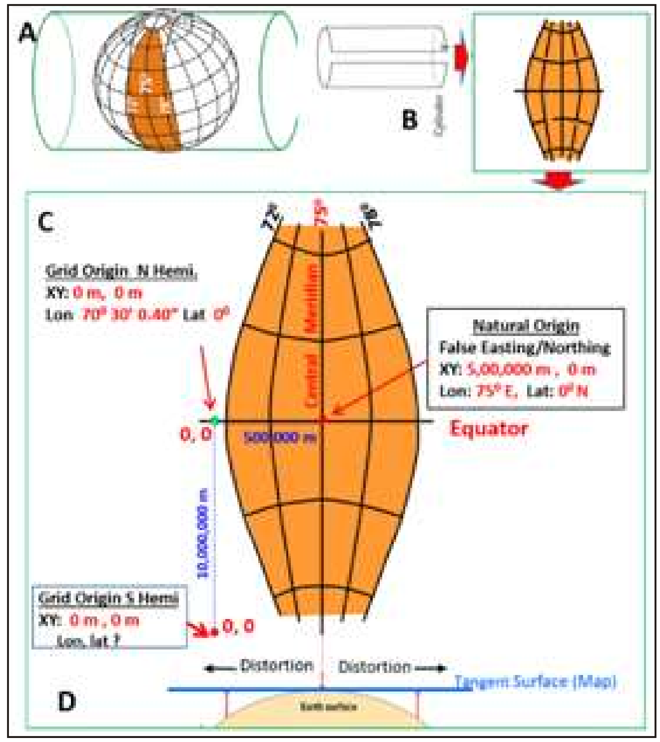


Fig. 8: UTM projection system.

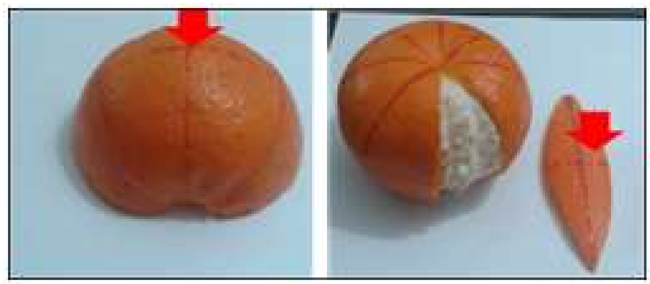
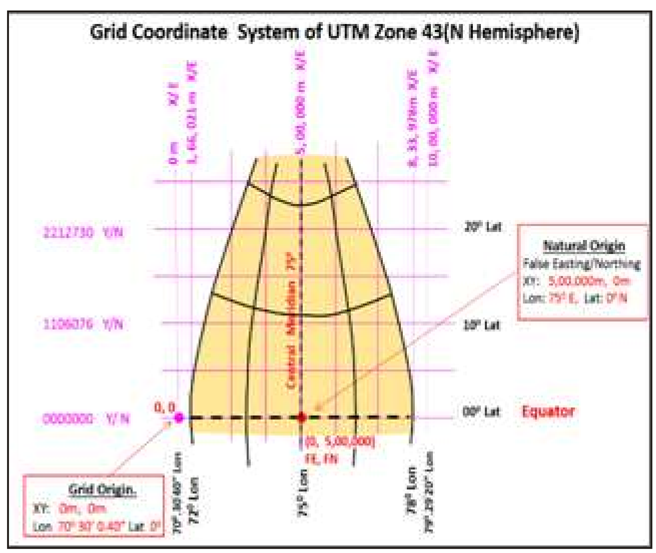


Fig. 9: Orange peel &UTM projection



In this example, the WGS 84/UTM43N zone, coordinate reference system has a natural origin (500000m E, 0 m N) lies at 0°N, 75°E.Its grid origin(0m E,0m N)) is at 0°N, 70°30'00.40"E.

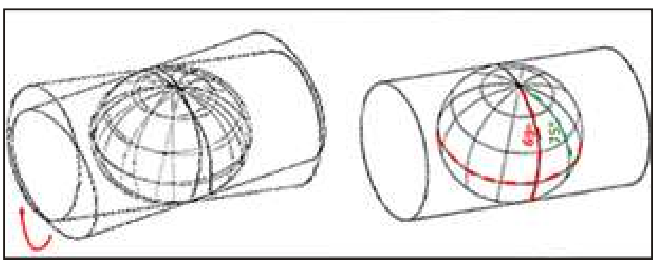
For mapping southern hemisphere (Fig.9C), natural origin at the equator is given a false northing of 10,000,000m, thus ensuring that no point in the southern hemisphere will take a negative northing coordinate.

The tangent meridian of the zone is known as the **Longitude of Origin (Central Meridian,** abbreviatedas **CM)** and provides the direction of the northing axis of the projected coordinate reference system.

Because of the steadily increasing distortion in the scale of the map with increasing distance from the central meridian, it is usual to limit the extent of a projection to 2 or 3 degrees either side of the central meridian. The cylinder, on rotating (Fig.11) clockwise, becomes tangent to the 69^0 meridian (new CM) and a new Zone 42 (CM 69) is created.

Similarly, 60 number of UTM zones, each 6 degrees wide, can be created, covering the ellipsoid between the 84 degree North and 80 degree south latitude. The different zones with their respective central meridians are shown in Fig.12. Each one has its own independent grid (X-Y/Northing-Easting) system. Thus, a value of X-Y is repeated sixty times. Figure also shows the corresponding UTM zones of Indian peninsula.

The resulting distortions due to the deformation of the surface will result in variation of scale throughout the flat map. Circle drawn on a spheroid, gets distorted in size and shape when projected onto a flat map (Tissot's indicatrices, Fig.13). In conformal case, the projection of circles vary substantially in size, but retain their circular shape. Cylindrical Transverse Mercator, Lambert Conformal Conic, and Polar Stereographic, among the most widely used conformal projections, are used for large scale surveying and mapping.



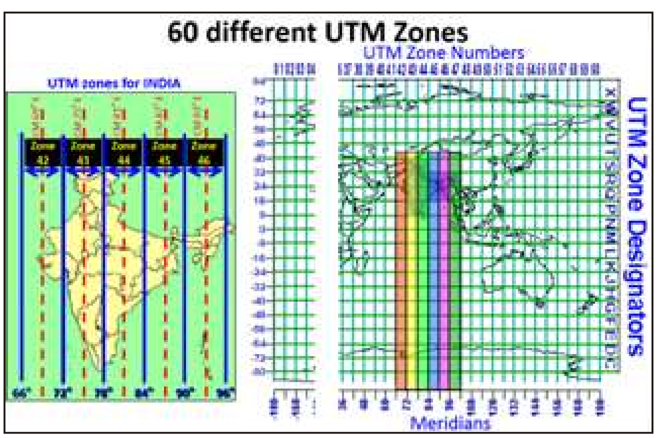


Fig. 12: UTM Zones

In case of equal area projection (Equivalent), circles are distorted, into ellipses, however of equal sizes. Equal area projections are preferred for small scale thematic mapping and other political maps.

Scale factor is the ratio of actual scale at a location on map to the principal (nominal) map scale (SF = actual scale / nominal scale). Along the line of tangency (CM), scale factor is 1 (zero distortion). Distortion increases (Fig.8D) with increasing distance away from the line of tangency (CM).

In order to further limit the scale distortion within the zone or projection area, some projections introduce a scale factor of 0.9996(rather than of 1)at the central meridian. It has the effect of reducing the scale in the major part of the map close to unity (Fig.14) along the CM. As a consequence, lines of unit scale factor (zero distortion lines) shift 180 km either side of the central meridian.

The essential parameters required to completely and unambiguously define Transverse Mercator Projections are:

1. Latitude of natural origin (Equator)
2. Longitude of natural origin (Central Meridian)
3. Scale factor on the central meridian (SF = 0.996)
4. False easting (FE)
5. False northing (FN)

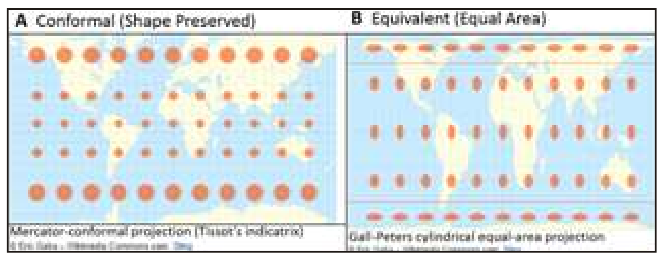


Fig. 13: Map distortion, Tissot's indicatrix

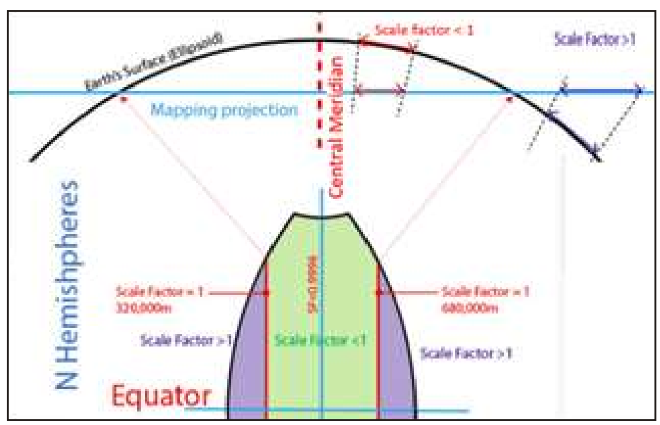


Fig. 14: Scale Factor in UTM Projection

Lambert Conic Conformal(LM) Projection

Imagine a paper cone placed over the Northern Hemisphere, tangent to a parallel (latitude), as shown in Fig.15A. The North Pole will be projected as a point at the apex of the cone. The meridians will radiate outward from the North Pole as straight lines. The parallels will appear as concentric circles, growing progressively smaller as latitude increases.

When the cone is cut (Fig.15B) along a meridian and flattened out, the meridians and parallels will appear as shown in Fig.15C. Similar to the previous orange analogy, a concentric strip of peel can be cut and flattened on to a plane (Fig.15D) to generate a Lambert's zone.

Cones with different cone angles (Fig.15E), each tangent to a different standard parallel, will generate different concentric strips (different lambert's zones).

For territories with limited latitudinal extent but wide longitudinal width, it may sometimes be preferred to use a single Lambert Conic Conformal projection rather than several UTM zones. But if the latitudinal extent is large, two or more zones need to be created to avoid the scale distortion.

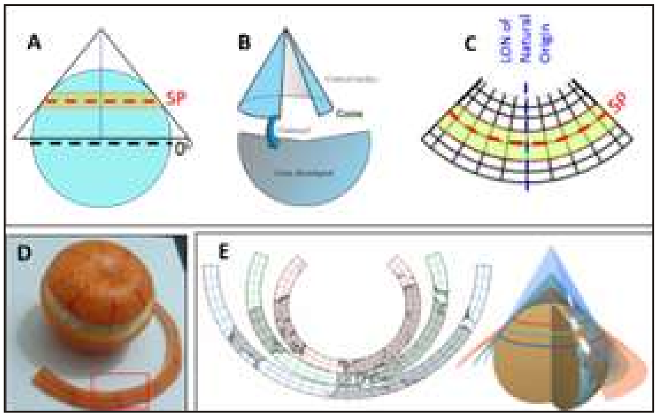


Fig. 15: Lambert Conic Conformal Projection system

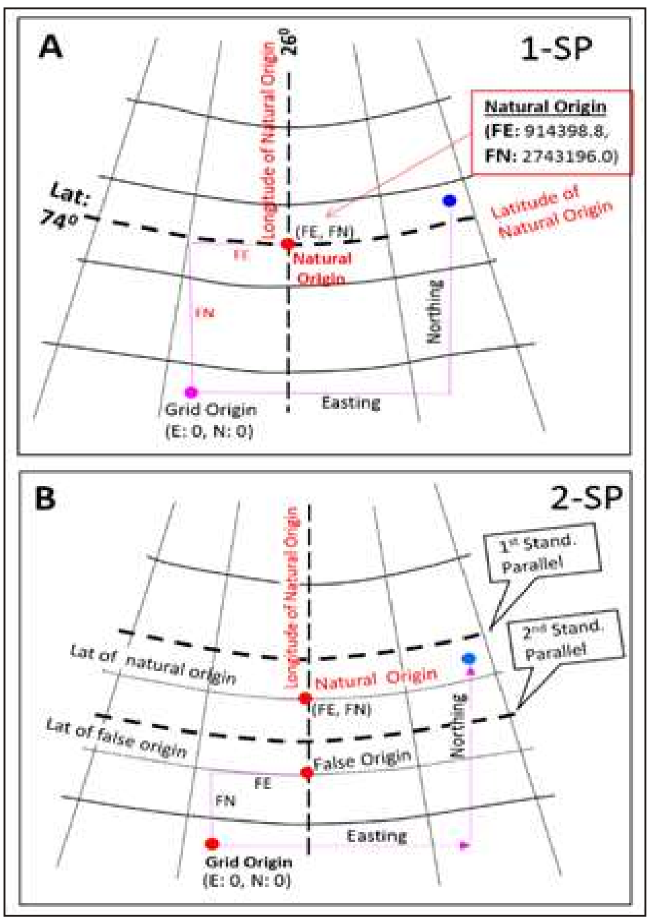


Fig. 16: Lambert Conic Conformal Projection,A:1SP, B:2SP

Conical projection with one standard parallel (**1-SP,** Fig.16A) is normally considered to maintain the nominal map scale along the parallel of latitude which is the line of contact (tangent) between the imagined cone and the ellipsoid. The natural origin of the reference system is the intersection of the standard parallel with the longitude of origin (central meridian). Tomainta in the conformal property the spacing of the parallels is variable and increases with increasing distance from the standard parallel, while the meridians are all straight lines radiating from a point on the prolongation of the ellipsoid's minor axis.

The distortion is minimum and the scale factor is unity along the standard parallel. Distortion increases both sides away from the parallel (Fig.17).

Sometimes however, the scale factor at the line of tangency is kept at less than unity for the same reason as with the case of UTM. It has the effect of making the scale factor unity on either side of the standard parallel. However, the two parallels obtained in this way do not have integer values of degrees or degrees minutes and seconds.

For the 1SP-LM, the essential parameters are:

1.	Latitude of natural origin:	26^0
2.	Longitude of natural origin:	74^0
3.	Scale factor at natural origin:	0.98554….
4.	False Easting:	914398.8 m
5.	False Northing:	2743196.0 m

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2. Shape (Conformal): Conformal projections preserve the relative local angles (shape preserved) about every point on the map, so that meridians intersect parallels at 90o.No map can be both equal-area and conformal
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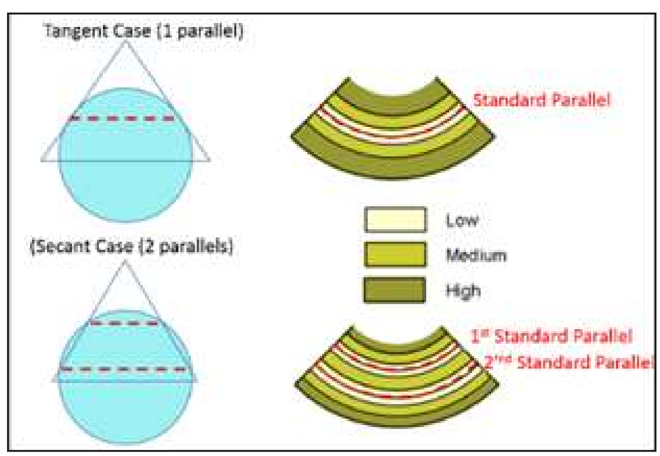

Fig. 17: Distortion in Lambert Conic Conformal Projection

4. Scale: No map projection shows scale correctly throughout the entire map; equidistant projections show true scale between two points or along every meridian.

In the present paper, we will be discussing cylindrical (Universal Transverse Mercator) projection and conical (Lambert Conic Conformal) projection type in detail which are relevant to oil industry in India. The planar (azimuthal) projection is used to map regions near to poles.

Lambert Conic Conformal(LM) Projection

Imagine a paper cone placed over the Northern Hemisphere, tangent to a parallel (latitude), as shown in Fig.15A. The North Pole will be projected as a point at the apex of the cone. The meridians will radiate outward from the North Pole as straight lines. The parallels will appear as concentric circles, growing progressively smaller as latitude increases.

The numerical values, given as an example, represent Cambay region where LM is used. Some literatures refer this as **zone IIa,** where Kalianpur datum and Lamberts Conformal Projection (1SP), is used as CRS.

In the **secant** case (**2SP**), the cone intersects or cuts through the earth as two circles, two standard parallels (Fig.16B). The two standard parallels (2SP)is usually chosen so that they each lie on the north and south margins of the mapped area. The distortion is minimum and the scale factor is unity along the two standard parallels (Fig.17). Features appear smaller between secant parallels and larger outside these parallels. Secant projections lead to less overall map distortion.

What happens when the projection parameters are misquoted?

Let us take an example of a fixed location on the earth with a defined latitude/longitude, having datum as WGS84. Projecting on a map under UTM system with different central meridian, the points falls apart by few tens of meter (40 to 160 m, Fig.18A). Keeping projection system same (UTM, CM 75), if datum is changed from WGS84 to Everest 1830, the location falls apart by thousands of kilometer (Fig.18B).

CRS transformation

A Lat-Long (Geographical Coordinates) of a location with a particular datum can always be transformed to WGS84 datum and subsequently to another datum. Projected coordinates (X-Y/Northing-Easting) need to be transformed first to its respective Geographical Coordinates (Lat-Long). A simple work-flow for coordinate transform is given in Fig.19.

Several methods exists for datum transformation. The principles of two methods are discussed below. In both the methods, the transformation aligns the x,y,z axes of the two datums in three-dimensional Cartesian coordinate space.

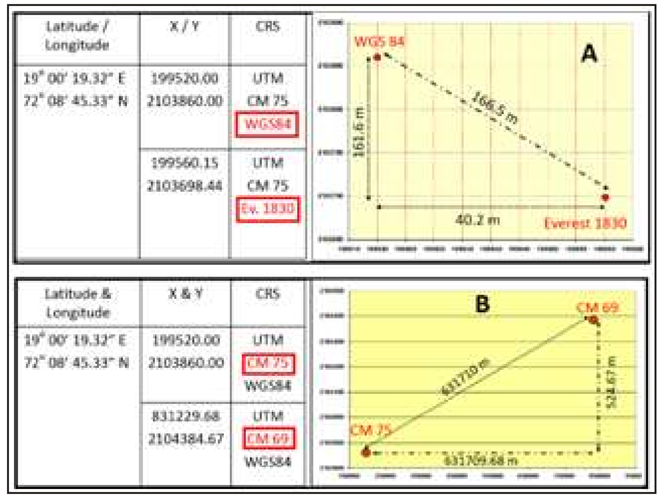

Fig. 18: Projection errordue to wrong CRS

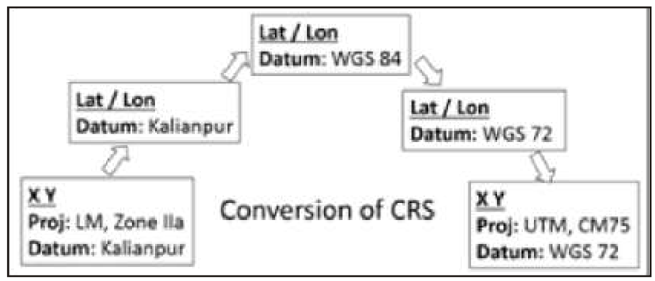

Fig. 19: CRS conversion

Three-parametermethods: In a three-parameter transformation (also called a geocentric translation), the axes of the two datums are aligned using linear shifts of the x, y, and z axes of the datum being transformed (Fig.20A). A three-parameter transformation is appropriate when the x, y, and z axes of the two datums are parallel and identically scaled. One datum is defined with its center at 0,0,0. The center of the other datum is defined at some distance (DX,DY,DZ) in meters away. Usually the transformation parameters are defined as going "from" a local datum "to" WGS 1984 or another geocentric datum. The Molodensky method converts directly between two geographic coordinate systems without actually converting to an X,Y,Z system. The Molodensky method requires three shifts (DX,DY,DZ) and the differences between the semi-major axes (Da) and the flattening (Df) of the two spheroids. The Projection Engine automatically calculates the spheroid differences according to the datums involved.

Seven-parameter methods: A seven-parameter transformation, a more complex and accurate datum

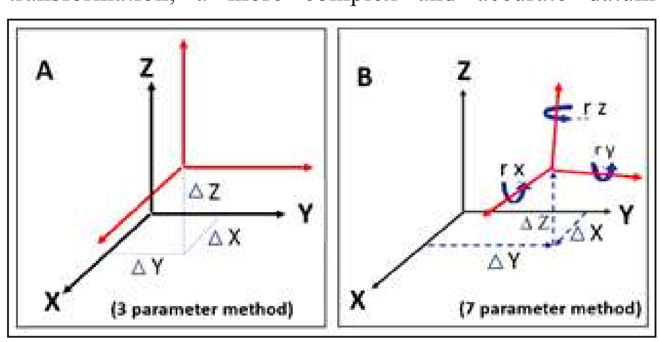

Fig. 20: Datum transformation methods

transformation, is used when the axes of the two datums are not parallel and identically scaled. In addition to the three linear shift parameters (DX,DY,DZ), three angular rotations around each axis (rx,ry,rz), and a scale factor are added(Fig.20B). The Molodensky–Badekas method is a variation of the seven-parameter methods.

Conclusion

The important lesson learnt through this discussion is that a point on the earth may have different Lat/Lon corresponding to different Datum. A Datum is defined by a Spheroid/ Ellipsoid which has a defined origin & orientation w.r.t the earth. Map projections are attempts to portray curved surface of the earth onto a flat surface with minimal distortion of geometrical properties, suitable for the purpose.

The World Geodetic System (WGS84) is a standard datum for use in cartography, geodesy, and navigation including GPS. One of the most widely used map projection system is UTM system which uses WGS84 as its datum. North Atlantic Treaty Organization (NATO) developed the Universal Transverse Mercator coordinate system which is recognized as highly accurate along Standard Parallel/Central Meridian, i.e. in N-S direction. Oil industries around the world have started using the global WGS 84 datum and UTM as projection system. However, for territories with limited latitudinal extent but wide longitudinal width, Lambert's Conic projection is preferred over Transverse Mercator to minimize distortion error.

It is utmost important to note that while reporting coordinates of a location, its complete CRS must be mentioned. **Geographic coordinates (Lat-Long) should bear its datum information. A map or data with projected coordinates of a location (X-Y or Northing-Easting) should always be accompanied by both datum and projection system including central meridian (standard parallel in case of LM).**

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