Uncertainty Analysis of Crustal Models Obtained From Inversion of Gravity Anomalies Using Genetic Algorithm

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ABSTRACT: We used Genetic Algorithm to invert gravity anomaly over the Nagaur-Jhalawar geotransect in Central India to delineate the crustal structure for three different models of a priori density distribution used by several workers. Our use of Bayesian statistics to compute the marginal a posterior probability density function of different crustal discontinuities in this region favors the values of densities for the upper crust, the lower crust and the mantle as 2700 kgm⁻³, 3100 kgm⁻³ and 3300 kgm⁻³ respectively. Further, the uncertainty analysis brings out clearly the regions in the transect where the structure is poorly constrained.

INTRODUCTION

Inverting gravity data to map major crustal discontinuities and other density boundaries is an important problem in crustal studies. In view of inherent ambiguity in gravity interpretation, unconstrained optimizations have limited applications in solving such problems. For any meaningful interpretation of gravity data, it is desirable to use as many a priori information as possible. Further, it is informative to specify the reliability of an interpreted model through its statistical attributes such as its probability density function, instead of representing the best-fit model. As one requires a large number of solutions to perform any statistical analysis, global optimizations techniques—those invariably deal with large number of models—are best suited to meet the objective. In this work, we used genetic algorithm to invert the gravity data over Nagaur-Jhalawar geotransect. We used a three-layered earth model comprising the upper crust, the lower crust and the mantle. We selected Talwani’s method as the forward solver due to its simplicity and robustness. Our objective function involves a weighted sum of misfit in gravity anomaly as well as its first derivative. This construction adapts to both smooth as well as sharp changes in the interface topographies. We analyzed the data with three sets of density information previously used by several researchers. Our analysis of uncertainty of the solutions—based on Bayesian statistics—favors a model with densities for upper crust, lower crust and mantle as 2700 kgm⁻³, 3100 kgm⁻³ and 3300 kgm⁻³ respectively.

METHODOLOGY

Genetic Algorithm, GA (Goldberg, 1989; Sen and Stoffa, 1995) works on the Darwin’s principle of ‘the survival of the fittest’. GA selects a large number of models—called population, at a specific instant. The models with higher fitness values have better probability to pass on their characters to the next generation. The entire process consists of three steps, viz., selection, crossover and mutation repeating with generations. With progressing evolution, models with larger fitness values contribute increasingly to the population. From numerical experiments on simulated examples, we observed that for gravity inversion for crustal studies, 30 - 50 generations are sufficient to obtain an acceptable result with a population size 100-200 models. For field data, around 100 generations are required to obtain satisfactory results especially when the number of model parameters is large.

We compute the gravity anomaly over a crustal structure using the method of Talwani et al. (1959) for 2D arbitrary shaped body assuming the density of each layer to be constant and known a priori. We represent each layer by a closed polygon with each of its horizontal side discretized by (n-1) equal intervals. We pose the inverse problem as that of determining the depths of these corners of each interface from a predefined wide parameter search space given the density of each layer. We adopted the following construction of the objective function

\[
\text{Fitness} = 1 - (w_1 \varepsilon_1 + w_2 \varepsilon_2), \ldots
\]

Where, \(w_1\) and \(w_2\) are two weights with the condition \(w_1 + w_2 = 1.0\).

The misfit functions \(\varepsilon_1\) and \(\varepsilon_2\) involving the gravity anomaly and its gradient respectively follow the work by Sen and Stoffa (1995)

\[
\varepsilon_1 = \frac{2\Sigma|g_{\text{obs}} - g_{\text{cal}}|}{\Sigma|g_{\text{obs}} - g_{\text{cal}}| + \Sigma|g_{\text{obs}} + g_{\text{cal}}|}
\]

\[
\varepsilon_2 = \frac{2\Sigma|S_{\text{obs}} - S_{\text{cal}}|}{\Sigma|S_{\text{obs}} - S_{\text{cal}}| + \Sigma|S_{\text{obs}} + S_{\text{cal}}|}
\]
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where ‘g’ stands for gravity anomaly and \( S^2 = (\delta g/\delta x)^2 + (\delta g/\delta z)^2 \). The subscripts ‘obs’ and ‘cal’ represent the observed and computed values respectively. The weights \( w_1 \) and \( w_2 \) depend on relative importance of the gravity anomaly and its derivative in mapping of an interface. We have selected \( w_1 = w_2 = 0.5 \).

RESULTS

**Nagaur – Jhalawar Geotransect, India.**

The Nagaur - Jhalawar geotransect runs for about 470 km perpendicular to the Aravalli mountains in the Central India with a NE-SW trend (Mishra et al., 1995). The Bouguer anomaly profile (Fig. 1) exhibits a conspicuous high in the central part, between 100 –250 km from Nagaur, known as the central gravity high (CGH) and a steep gradient on NW segment where as a gentle slope in the SE part. At a first impression this indicates a sharp faulting in the density interface beneath NW side and smooth variation in the SE part. Several workers interpreted the gravity data in this region to delineate the crustal structure. Ramakrishna (1988) proposed uplifting of the lower crust from around 20 km to a depth as shallow as 6 to 12 km to explain the CGH. He assumed a density contrast of -200 kg/m\(^3\) between the lower crust and the mantle. Mishra et al. (1995) computed the average depths of Conrad and Mohorovicic discontinuities from spectral analysis. They modeled the anomaly by assuming densities of the upper crust, lower crust and mantle as 2700, 3100 and 3300 kg/m\(^3\) respectively. They selected high density for the lower crust to explain under plating of the mantle material in the overlying crust (Sharma, 1995). A deep seismic survey (Tewari et al.,1995; Reddy et al., 1995) revealed a deeper Moho in Delhi fold belt region, i.e., between 120 km and 170 km from Nagaur. We have discretized the gravity anomaly at an interval of 5 km. We selected the range of depth variation for the shallow and the deep interfaces to be between 8-25 km and between 30 - 45 km respectively.

We describe below the results of inversion of the observed gravity data by GA and the corresponding analyses of uncertainty in the interpreted crustal structures for three different density models considered by previous workers.

**Model 1** : Figure 1a shows the results of inversion with density of the upper crust, the lower crust and the mantle as 2700 kg/m\(^3\), 2900 kg/m\(^3\) and 3300 kg/m\(^3\) respectively. Figure 1b presents the crustal structures obtained with and without using derivative information. It clearly shows that some sharp edges of the model are delineated after using the derivative information. The derived model shows a thickening of the lower crust below the CGH. A steep fault bounds northwestern part of the CGH that controlled the deposition of Marwar group of sediments. Further, the crust towards Jhalawar thickens below the traps.

![Figure 1](image1.png)

**Model 2** : In this case the values of densities for three layers are 2700 kg/m\(^3\), 3000 kg/m\(^3\) and 3300 kg/m\(^3\). The results (Figure 2) are similar to the previous case excepting for smoothing of the interfaces in the southeastern part of the profile.

![Figure 2](image2.png)

**Model 3** : Figure 3 represents the results obtained for the density model, in which the values of densities for the upper crust, lower crust and mantle are 2700 kg/m\(^3\), 3000 kg/m\(^3\) and 3300 kg/m\(^3\) respectively. The derived models show much smoother variation in the interfaces compared to the results of the previous cases. The shallow interface shows a steep fault at the northwestern boundary of CGH. The lower crust shallows up to nearly 10 km from the surface. Down warping of Moho below CGH and adjoining region results in the thickening of the lower crust.

![Figure 3](image3.png)
we formulate the problem of appraisal of the solutions through statistical analyses of the models showing data fit better than an acceptable threshold value. Following a method by Sen and Stoffa (1996), we construct an approximate marginal posterior probability density function (PPD) for each parameter. To meet this objective, we divide the model space into small grids and calculate the probability of occurrence of a corner of the interface on a particular grid. Figures 4, 5 and 6 shows the plots of PPD for model1, model2 and model3 respectively. It is interesting to note that the best fitting models of Figures 1, 2 and 3 and the corresponding models constructed by joining the maximum values of PPDs in Figures 4, 5 and 6 respectively exhibit striking similarity in the gross crustal structures. Further, from spatial spreading of PPD plots we can infer that among the three different a priori density models the uncertainty associated with model 3 is the least as it shows narrower vertical spread than the remaining two cases. Further, we observe an interesting distribution of PPD around 350 km from Nagaur for all three models, viz. high values of PPD are present at several depths. This indicates presence of greater uncertainty in model parameters in this part of the profile.

CONCLUSION

We have used genetic algorithm to invert gravity anomalies for delineating regional density interfaces. GA does not require any starting model; rather it selects a large number

Figure 3: Inversion of gravity data along Nagaur-Jhalawar geotransect. Densities of upper crust, lower crust and mantle are 2700 kg/m 3, 3100 kg/m 3 and 3300 kg/m 3 respectively.

Figure 4: Distribution of posterior probability density function for the inverted models with prior density distribution as shown in figure.

Figure 5: Distribution of posterior probability density function for the inverted models with prior density distribution as shown in figure.
of models from a wide parameter space. Instead of a single solution, GA determines several probable solutions that we used to derive statistical attributes of the inverted models. The use of derivative in objective function facilitates better delineating sharp edges and faults. The uncertainty analysis of the inverted models over Nagaur-Jhalawar geotransect favors density values for upper crust, lower crust and mantle as 2700 kg/m\(^3\), 3100 kg/m\(^3\) and 3300 kg/m\(^3\) respectively.

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