Single Frequency Seismic Attribute Based on Short Time Fourier Transform, Continuous Wavelet Transform, and S Transform

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Summary

Spectral decomposition is an important signal analysis tool for seismic data. The overall frequency content of a signal can be obtained from Fourier transform. However, for a non-stationary signal, such as seismic signal, whose frequency content varies with time, 1D transformation in frequency is not sufficient. Traditionally, 2D representation in time and frequency space for a 1D signal is achieved by taking the Fourier transform over a short-time window. This method is commonly known as short-time Fourier transform (STFT). Time-frequency resolution in STFT is limited by the choice of a window length. The windowing problem in time-frequency analysis is absent in continuous wavelet transform (CWT) method. S transform is a phase corrected form of CWT that yield a good result in analysis of seismic signals.

Introduction

Spectral decomposition provides a tool for imaging and mapping temporal bed thickness and geological discontinuities over large 3-D seismic surveys (Gridley and Partyka, 1997; Partyka et al., 1999). This technique has been used to delineate facies and depositional environments such as channel sands and incised valley-fill sands (Peyton et al., 1998). Spectral decomposition typically employs the discrete Short Time Fourier transform to generate mono-frequency images from the broadband seismic data. However, this approach obviously has some short comes because of the type and the length of the selected window and significant properties of the seismic signals must be ignored. Windowed Fourier transform-based methodologies bias the amplitude spectra. The wavelet transform allows a seismic signal to be examined in both the time and frequency domains simultaneously. It has become a popular tool for the analysis of non-stationary signals and has replaced the conventional Fourier transform in many practical applications (Van den Berg, 1999).

We introduce an instantaneous spectral imaging technique by using wavelet transform and S transform and the results are compared with that of the Short Time Fourier Transform. The results of our work and previous works (e.g. Sinha, 2004; Sun and Castagna, 2002) prove that instantaneous spectral analysis can achieve excellent time and frequency localization while avoiding windowing problems.

Attenuation of seismic waves traveling through oil and gas reservoirs is a function of reservoir rock properties. This attenuation is observed as a pronounced loss of high frequency energy, and these attenuation anomalies can be a useful hydrocarbon indicator. Since instantaneous spectral analysis allows us to obtain frequency spectrum at each time sample for a seismic trace, seismic attenuation can be described as a frequency-dependent spectral variations. These variations enable us to detect attenuation of the high-frequency components associated with oil and gas reservoirs.

The principal aim of this paper is to compare the spectral analysis potential of the continuous wavelet transform and S transform in order to direct hydrocarbon detection and thin layer imaging on seismic sections.

Short time Fourier Transform

The instantaneous frequency has often been considered as a way to introduce frequency dependence on time. If the signal is not narrow-band, however, the instantaneous frequency averages different spectral component in time. To become accurate in time, we therefore need a two dimensional time-frequency representation of the signal $x(t)$composed of spectral characteristics depending on time. By assuming that signal is stationary when seen through a window g(t) of limited extent, centered at location τ, the Fourier transform of the windowed signals, $x(t)g^*(t-\tau)$ yields the Short Time Fourier Transform (Rioul and Vetterli, 1991):

$$STFT(\tau, f) = \int x(t) g^*(t-\tau) e^{-j2\pi ft} dt$$

Which maps the signal into two dimensional function in (τ, f) plane. Location of time-frequency window is determined by τ and f. In time-frequency analysis we want to have a window as small as possible to achieve a better resolution. On the other hand, Heisenberg uncertainty principle limits the window's area (Zabihi, 2005)

$$\Delta t \Delta f \geq \frac{1}{4\pi}$$

It means that one can only trade time resolution for frequency resolution, or vice versa. More important is that once the window has been chosen for the STFT, then the time-frequency resolution is fixed over the entire time-frequency plane, which is shown in fig. 1a.

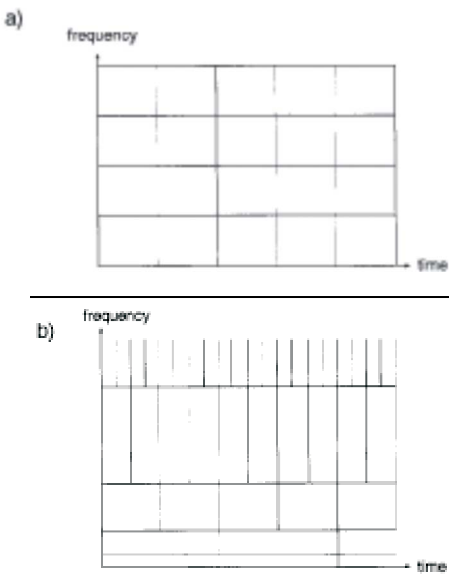

Fig. 1. Time-frequency resolution of the a) STFT and b) CWT.

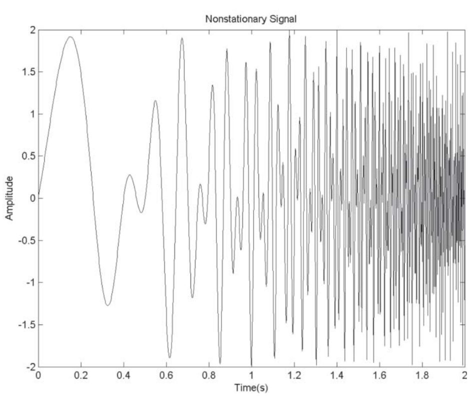

Fig.2. Synthetic signal.

Continuous wavelet transform

As mentioned above the STFT requires a fixed time support. In practice, seismic data are nonstationary and the STFT may not produce very reliable time-frequency map of it. Fixed window length and hence, fixed time-frequency resolution is a fundamental difficulty with the STFT for analyzing of a non-stationary signal.

Continuous wavelet transform was introduced by Morlet et al. (1982). Continuous wavelet transform (CWT) is an attempt to produce better time-frequency map. In CWT, time-frequency atoms are chosen in such a way that its time support changes for different frequencies honoring Heisenberg's uncertainty principle (Mallat, 1999; Daubechies, 1992). Such a time-frequency atom is called a wavelet.

A wavelet is defined as a function $h(t)$ with a zero mean, which is localized in both time and frequency.

$$\int_{-\infty}^{\infty} h(t) dt = 0$$

Continuous wavelet transform of signal $x(t)$ is given by:

$$CWT_x(\tau, a) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} x(t) h^*\left(\frac{t-\tau}{a}\right) dt$$

Where $h^*(t)$ is complex conjugate of , a is scale and $\frac{1}{\sqrt{|a|}}$ is for energy normalization. For the inverse transform to exist it is required that the analyzing wavelet satisfy the "admissibility condition" (Daubechies, 1992). A commonly used wavelet in continuous wavelet transform is Morlet wavelet that provides an easy interpretation from scale to frequency. It can be shown that scale in Morlet wavelet is inversely proportional to frequency (Zabihi, 2005), and also in general we can convert time-scale map to time-frequency map by (Sinha, 2004):

$$X(\omega, \tau) = \frac{1}{c} \int_{-\infty}^{\infty} CWT_x(\tau, a) e^{-j\omega\tau} H(a\omega) \frac{da}{a^{3/2}}$$

where c is wavelet dependent constant and $H(a\omega)$ is the scaled version of wavelet in frequency domain. Adaptive window in continuous wavelet transform has dimension $(a\Delta t, \frac{\Delta a}{a})$ in time-frequency domain. So CWT based time-frequency map has higher frequency resolution at lower frequency and higher time resolution at higher frequencies that is shown in fig.1b. In addition to improvement in resolution the introduced methodology does not depend on the choice of the window length.

S Transform

There are several ways of arriving at the S transform. We consider the S transform as a "phase correction" to the continuous wavelet transform (Stockwell et al., 1996). Written out explicitly, the S transform of signal

$x(t)$ is

$$S(\tau, f) = \int_{-\infty}^{\infty} x(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(\tau-t)^2 f^2}{2}} e^{-j2\pi ft} dt$$

Where the mother wavelet is defined as

$$h(t, f) = \frac{|f|}{\sqrt{2\pi}} e^{-\frac{t^2 f^2}{2}} e^{-j2\pi ft}$$

This wavelet does not satisfy the condition of zero mean for an admissible wavelet and also because the oscillatory parts of the S transform "wavelet" is provided by the complex Fourier sinusoid, which does not translate with the Gaussian window when τ is changed. As a result, the shape of the real and imaginary parts of the S transform "wavelet" change as the Gaussian window translates in time. True wavelets do not have this property because their entire waveform translates in time with no change in shape. Thus, the *S*-transform is conceptually a hybrid of short-time Fourier analysis and wavelet analysis, containing elements of both but falling entirely into neither category (Pinnegar and Mansinha, 2003).

Figure 2 shows a nonstationary signal and figure 3 shows a time-frequency power spectrum of STFT, CWT and S transform for this synthetic signal. We see that because of nonstationarity nature of signal STFT can not detect the entire frequency band in time with a given window length. CWT determine entire frequency band in time with lower amplitude than the S transform. We see that Time-frequency resolution of the S transform is similar to the wavelet transform except a bit difference at the end of frequency band.

Single frequency seismic section

Mapping of a seismic trace in to the time-frequency domain produces a two dimensional data set by adding a frequency axis. In a similar way a two dimensional seismic section will generate a 3D data cube in which the third axis being frequency up to the Nyquist frequency. Any section at a single frequency from the 3D data cube is called a *single frequency seismic* (SFS) section. Comparison of different SFS sections can be utilized to detect low frequency shadows caused by presence of the hydrocarbon reservoirs. This method can potentially be utilized as a direct hydrocarbon detection tool (Sinha, 2004).

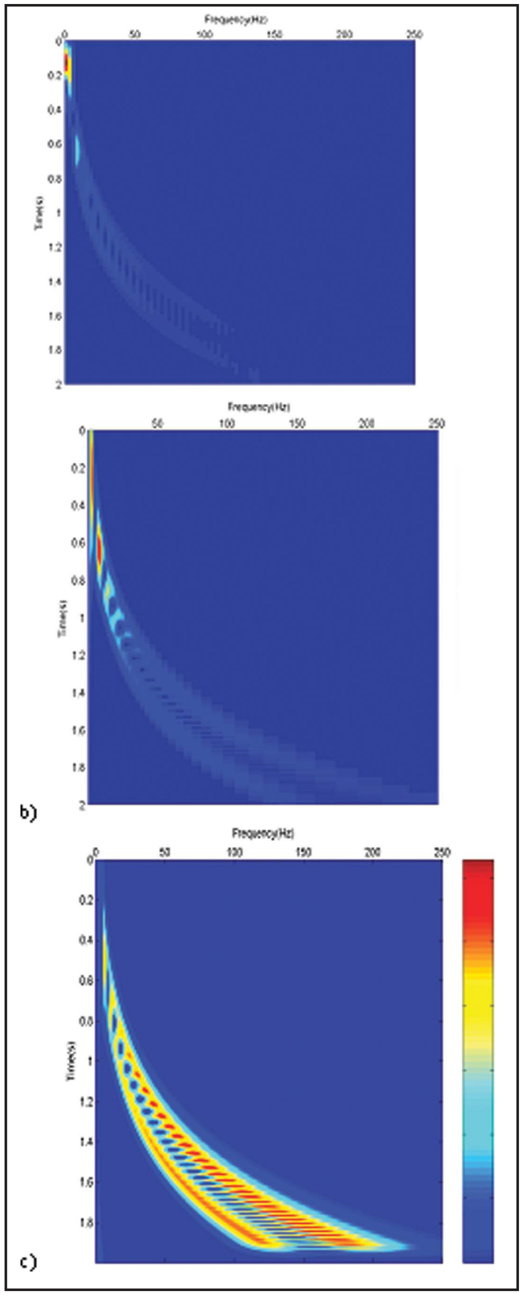

Fig.3. Time-frequency power spectrum of a) STFT b) CWT c) S transform

We applied the methods on a seismic section with a known hydrocarbon reservoir location. We compared single frequency seismic section of the STFT, CWT and S transform and discussed the differences between the results of the methods. Figure 4 shows single frequency seismic section

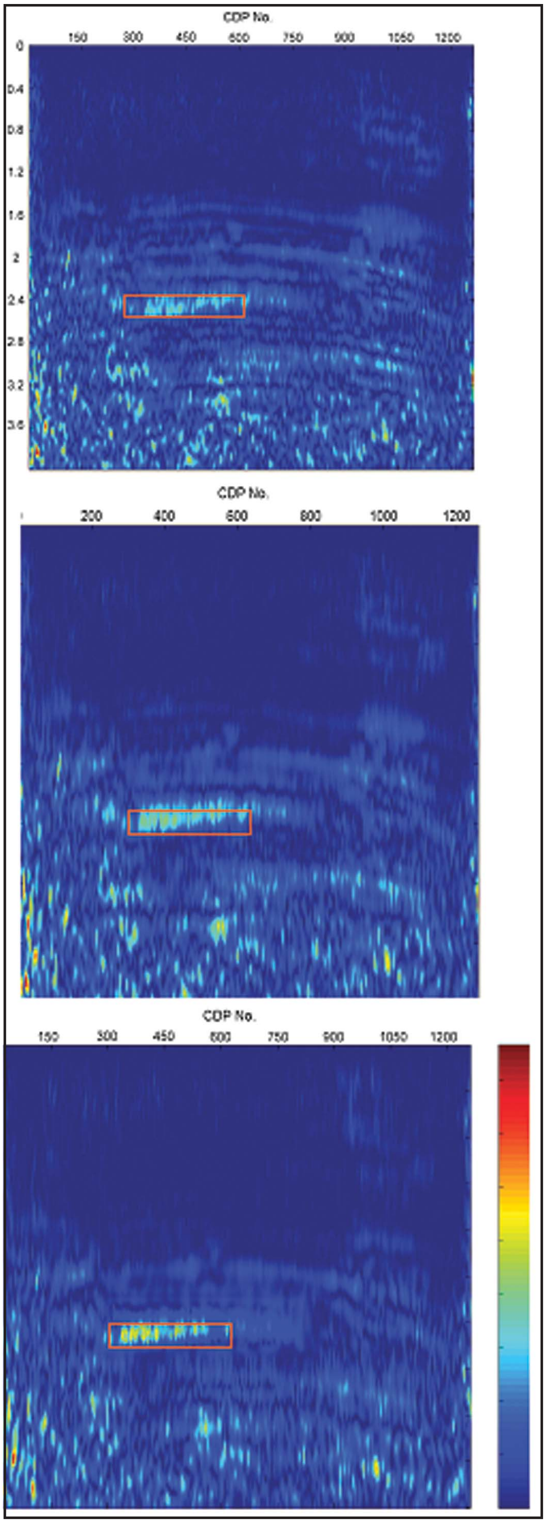

Fig. 4. Single frequency seismic section derived by a) STFT b) CWT c) S transform. CWT and S transform yield the similar result but with different amplitude.

at frequency 15 Hz. Low frequency anomalies is highlighted below the reservoir location and disappeared at higher frequency SFS sections.

Conclusion

In seismic industry, attribute analysis plays an important role in obtaining enhanced information about subsurface (Hart, 1999; Tanner and Sheriff, 1977). A commonly used technique in attribute analysis is spectral decomposition based on short time Fourier transformation but the spectral decomposition using STFT has inherent drawbacks. Wavelet based method solve some of these drawbacks. CWT has advantages like adjustable window, optimal time-frequency resolution and obtained better attribute. A problem with this method is conversion of scale to frequency to make the interpretation easier but this takes a computer time. Morlet wavelet does not have this problem seriously but S transform solve the problem completely because it achieved the Fourier frequency directly. Both of them (CWT and S transform) have a similar time-frequency resolution but different properties in their achievement.

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