True Amplitude Migration-Inversion in Phase Space

Mrinal K. Sen*, Paul L. Stoffa, Roustam K. Seifoullaev John A. ,Katherine G. Jackson
School of Geosciences, The University of Texas at Austin, USA

Summary

We have developed a new method for pre-stack depth migration in laterally varying media possibly including anisotropy in a coupled ray-parameter domain. The conventional Kirchhoff migration is carried out in configuration space in which ray-theoretical Green's functions are used to derive the so-called migration weights in a true amplitude migration. Our formulation makes use of Chapman-Maslov Green's function which results in a migration operator in the phase-space. The most general development includes both source and receiver Green's functions in the mixed phase-space of ray-parameter and depth. For application to seismic data migration, this dictates the use of data transformed to source and receiver ray-parameter and delay-time domain using double slant stacks. Although Maslov asymptotic theory overcomes some of the shortcomings of asymptotic ray theory, the principal advantage of our formulation for migration results from significant reduction of data volume and increase in computation speed. Wavefields that are propagated through the model are parameterized by surface source/receiver rayparameters. We make use of Eikonal equation based travel time algorithm to evaluate the integrand of the migration operator. The resulting algorithm is computationally very efficient and thus can be used for velocity model building.

Introduction

Seismic amplitudes are affected by several factors including geometrical spreading, reflection and transmission coefficients (reflectivity), source/receiver directivity, focusing and defocusing (part of spreading), anisotropy, attenuation and possibly some other factors. Of these, the reflectivity plays the most important role in that it serves as a proxy to lithology and hydrocarbon. Although traditional seismic imaging algorithms are aimed at obtaining correct estimates of reflector locations, more recent algorithms of true amplitude migration have larger goals - the goal is to derive migrated gathers in which amplitude variation is caused by reflectivity only. As such the very definition of true amplitude is related to a 'model' and thus we can, at best, expect to estimate true relative amplitudes.

Although attempts have been recently made to derive true amplitude migration operator for wave equation migrations, majority of true amplitude migration operators reported in the literature are asymptotic and apply to Kirchhoff migration. Derivation of Kirchhoff integral operators for the forward problem is straightforward; it involves using Green's theorem and often makes use of asymptotic ray theory Green's function. Inherent to the development of Kirchhoff forward modeling operator is a 'tangent plane hypothesis' or a Kirchhoff approximation that assumes that the wavefronts are locally planar at the scatterer and they interact with imaginary planes defined by a normal at the local scatterer comprising a reflector.

A true amplitude Kirchhoff migration requires development of an operator which is applied to surface measured data to estimate band-limited reflectivity. Development of such an inverse operator is not trivial, however. There are two basic approaches. One development maps the Kirchhoff integral into a generalized Radon transform integral and then makes use of inverse Radon transform to derive the inversion or true amplitude migration formula (Beylkin 1985). It is worthwhile to mention that several approximations (e.g, slowly varying amplitude field etc) are made while solving the inverse transformation analytically. Another approach is to directly employ an imaging condition (ratio of upgoing and downgoing wavefields) in the derivation of true amplitude migration formula (Keho and Beydoun 1988). It can be shown that the two disparate developments arrive at nearly identical formulas for true amplitude Kirchhoff migration (Docherty 1991).

Here we make use of the Kirchhoff integral but replace the asymptotic ray-theortical (ART) Green's function with Chapman-Maslov Green's function resulting in a migration operator that is highly efficient.

Theory

In the frequency domain the Kirchhoff integral (Stolt, 1978; Schneider, 1978) for wavefield continuation of sources and receivers to depth is

$$P(x,\omega) = \int \partial_n G(x,s,\omega) ds \int \partial_n G(x,r,\omega) P(s,r,\omega) dr$$

where $P(s,r,\omega)$ is the seismic wavefield measured at the where surface, G is the Green's function, $\partial_n G$ is the surface normal derivative of the Green's function, x is the subsurface location and P(x,ω) is the predicted wavefield at depth wavefield at depth. To extrapolate the measured seismic wavefield $P(s,r,\omega)$ we need to construct the Green's function. Conventional Kirchhoff migration makes use of ART Green's function.

In a homogeneous medium we can express the Green's function in terms of plane waves and derive a Kirchhoff integral in the plane wave domain (Sen and Frazer, 1991). For in homogeneous media we make use of Chapman-Maslov asymptotic theory (Chapman, 2004; Chapman and Drummond, 1982) in which the Green's function is given by

$$G(\mathbf{x},\mathbf{s},\omega)=\omega^2\int A_s(\mathbf{x},\mathbf{p}_s)\exp\left(i\omega t(\mathbf{p}_s,\mathbf{x})\right)d\mathbf{p}_s,$$

where the integration is carried over rays characterized by a parameter p. In our application p is the horizontal slowness at the surface. The amplitude term is given by

$$A_s(x,\mathbf{p}_s)=\left|\frac{dp_{11}}{d\mathbf{p}_s}\right|^{1/2}\exp\left(i\frac{\pi}{4}\left(\operatorname{sgn}\left(\frac{dp_{11}}{d\mathbf{p}_s}\frac{dx_{11}}{d\mathbf{p}_s}\right)-1\right)\right)$$

where $\hat{p}_{11}\cdot\hat{n}=0, \hat{x}_{11}\cdot\hat{n}=0.$ The phase term is given by (see figure 1)

$$t(\mathbf{x},\mathbf{p}_s)=\tau(\mathbf{x},\mathbf{p}_s)+\mathbf{p}_s\cdot(\mathbf{s}-\xi)$$

Similar expressions can be written down for G (x,r, ω) and we obtain

$$P(\mathbf{x},\omega)=-\omega^6\iint A_s(\mathbf{x},\mathbf{p}_s)\partial_n t(\mathbf{x},\mathbf{p}_s)A_r(\mathbf{x},\mathbf{p}_r)\partial_n t(\mathbf{x},\mathbf{p}_r)$$
$$\exp\left(i\omega(\tau(\mathbf{x},\mathbf{p}_s)+\tau(\mathbf{x},\mathbf{p}_r)-(\mathbf{p}_s+\mathbf{p}_r)\cdot\xi)\right)P(\mathbf{p}_s,\mathbf{p}_r,\omega)d\mathbf{p}_s d\mathbf{p}_r.$$

Ignoring filter ω^6, after summing over all frequencies, we get

$$P(x)=-\iint A_s(\mathbf{x},\mathbf{p}_s)A_r(\mathbf{x},\mathbf{p}_r)\partial_n t(\mathbf{x},\mathbf{p}_r)$$
$$P(\mathbf{p}_s,\mathbf{p}_r,\tau(\mathbf{x},\mathbf{p}_s)+\tau(\mathbf{x},\mathbf{p}_r)-(\mathbf{p}_s+\mathbf{p}_r)\cdot\xi)d\mathbf{p}_s d\mathbf{p}_r.$$

If now we define

$$L(\mathbf{x},\mathbf{p}_s,\mathbf{p}_r)=-A_s(\mathbf{x},\mathbf{p}_s)A_r(\mathbf{x},\mathbf{p}_r)\partial_n t(\mathbf{x},\mathbf{p}_s)\partial_n t(\mathbf{x},\mathbf{p}_r),$$

We have

$$P(\mathbf{x})=\iint L(\mathbf{x},\mathbf{p}_s,\mathbf{p}_r)P(\mathbf{p}_s,\mathbf{p}_r,\tau(\mathbf{x},\mathbf{p}_s)+\tau(\mathbf{x},\mathbf{p}_r)-(\mathbf{p}_s+\mathbf{p}_r)\cdot\xi)d\mathbf{p}_s d\mathbf{p}_r,$$

which is the true amplitude migration. This result is similarm to the one reported in Stoffa et al. (2005) except for the amplitude term. At this stage we note two salient features of our development

- Most true amplitude Kirchhoff migration operators in the configuration space assume that the amplitude factors are slowly varying and are taken out of the intgrand. This is not necessary in our development.
- Plane wave formulations are generally valid in 1D media or for dipping layers (Stoffa et al. 1981). In our development Ps and Pr are simply used as parameters to define the wavefields which can be propagated through the laterally varying models using either a ray method (e.g. Eikonal solver as used here) or by a one-way wave propagator.
- Chapman-Maslov operators are available for anisotropic media (Chapman 2004) with the only additional requirement of evaluating vertical delay times in such media. Sen and Mukherjee (2003) and Mukherjee et al. (2005) give a detailed outline of deriving these.

Examples

The examples are based on a 2D staggered grid elastic finite difference simulation, (Levander, 1988) for the EAEG salt data, see Figure 2. The data were acquired every 20m along the top of the model for 675 shot positions. The acquisition proceeded from the left (X=0.0km) to the right (X=13.48 km). We simulated a marine survey with a receiver array towed behind the ship. 240 channels were acquired with the first complete shot gather occurring at shot point 240 (X=4.78 km). The receiver spacing was 20 m. The first layer was water and only pressure was recorded. Absorbing boundaries were added to the model to limit reflections from the edges and bottom of the model and to minimize surface related multiples. For example shot records from the middle of the survey and over the salt are shown in Figure 3.

Results

The original shot gather data were transformed into the conventional offset plane wave domain by simple slant stacking. 121 plane wave seismograms for ray parameters +0.6 to -0.6 sec/km every 0.01 sec/km were recovered from the input shot gathers. The origin was taken relative to each shot's position and the plane wave gathers of Figure 4 correspond to the common shot gathers of Figure 3.

The original data were also simultaneously transformed to construct both source and receiver plane waves using equation (1). This process completely transforms the data into plane wave components. The

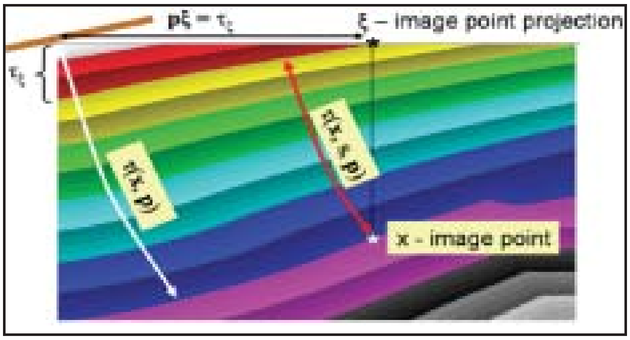

Fig.1: Isochrons and plane wave vertical delay time

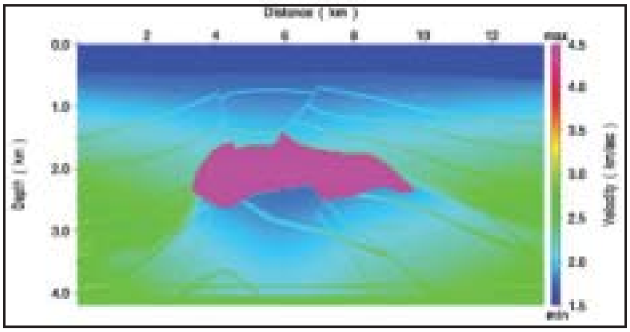

Fig. 2 : EAEG salt model

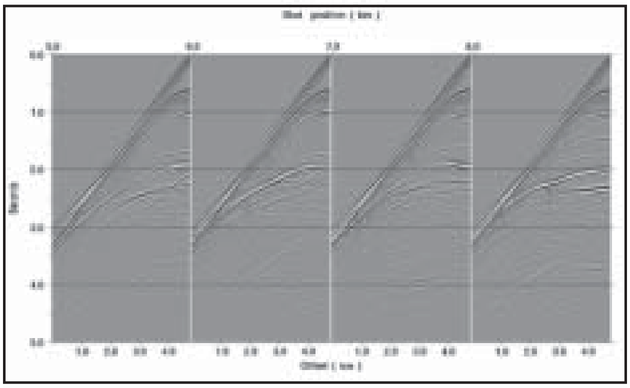

Fig.3 : Finite difference common shot gathers at source positions 5, 6, 7 and 8 km simulating a marine survey with the array towed behind the ship. 240 channels were acquired with a receiver spacing 20m. The maximum offset is 4.78 km

appearance of this reduced data volume is not easy to interpret so we show several cuts through the volume in Figure 5, for all pr plane waves for the cases where ps = -0.2, ps = 0.0 and ps = 0.2 sec/km from left to right in three panels.

We also transformed the data to source and offset plane waves. Figure 6 shows the case for all po plane waves for the cases where $p_s = -0.5$, $p_s = 0.0$ and $p_s = 0.5$ sec/km from left to right in three panels. Here the *ps = 0.0* (center) gather corresponding to horizontal reflectors dominates the others and appears similar to a conventional single shot τ- p_o gather. Figure 7 shows the opposite case, for $p_o = 0.0$ sec/km and all source plane waves.

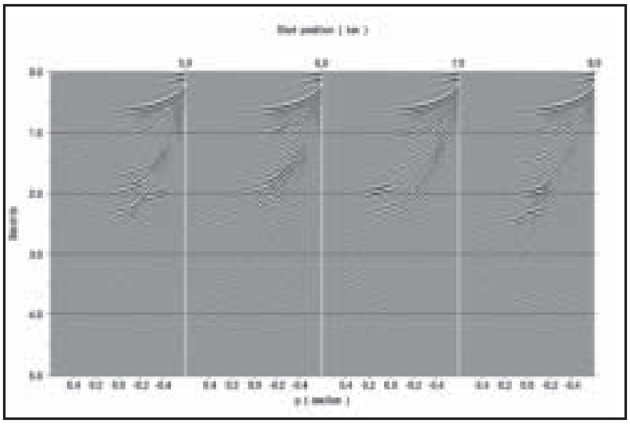

Fig. 4 : τ-p transformed shot point gathers at source positions 5, 6, 7 and 8 km. 121 traces in each panel correspond to ray parameters from +0.6 to -0.6 sec/km every 0.01 sec/km

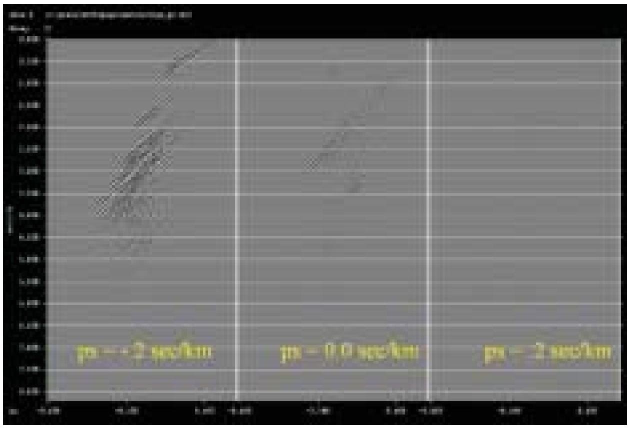

Fig.5: *ps* cross sections from *ps – pr* volume

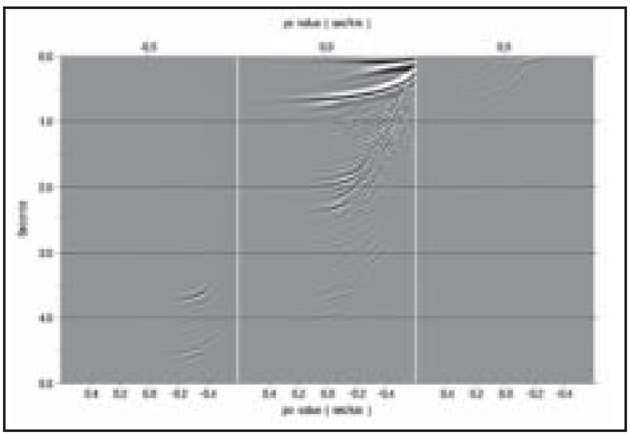

Fig. 6: *ps* cross sections from *ps – po* volume

The p_s–p_o volume was migrated using equation (10)

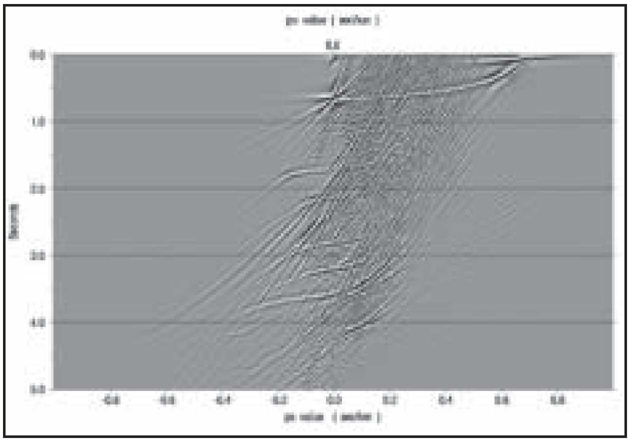

Fig.7 : *po* =0.0 cross section from *ps – po* volume

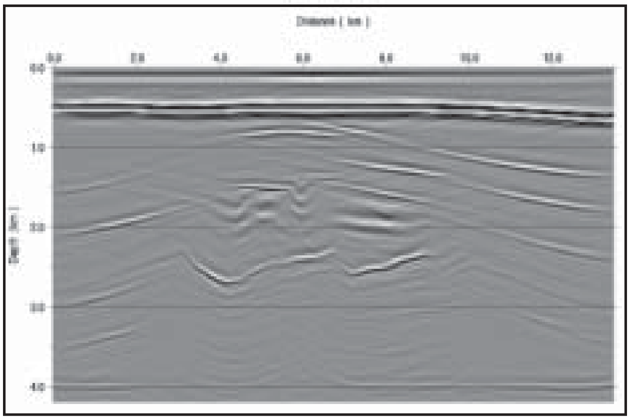

Fig. 8: *ps – po* migrated shot gather: *ps* values range from -0.1 to 0.1 sec/km, *po* values range from - 0.6 to 0.6 sec/km

and an eikonal solver, see Schneider et al., 1992, to calculate the vertical delay times. Each constant offset ray parameter plane wave section was migrated independently of the others and in parallel. Once all plane wave sections were migrated, the resulting common image gathers were stacked to generate the final image.

Plane wave vertical delay times were reused once computed as appropriate. For example, vertical delay times for any p, whether p_r, p_o or p_s, can be reused whether we need a p_s, p_r or a p_o as long as it has previously been computed.

Figure 8 shows the result for a targeted imaging where we used all 121 po plane waves but limit the ps aperture to - 0.1 to +0.1 sec/km about each po plane wave being imaged. This means that we are imaging principally reflection data. Figure 8 has a low spatial frequency appearance since only reflections are imaged. This approach is useful for velocity analysis as the imaging is computationally very fast and we can add more ps aperture

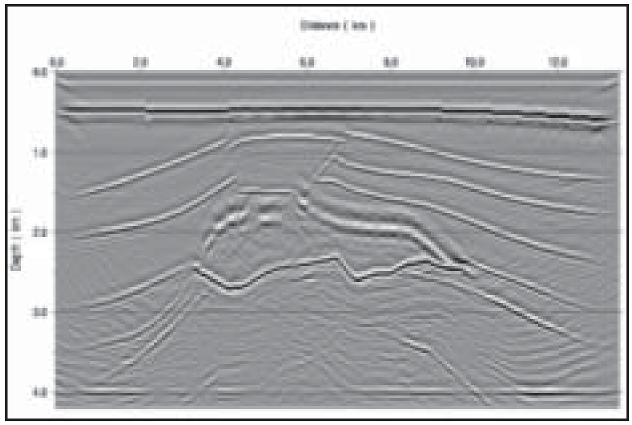

Fig.9 : *ps – po* migrated shot gather: *ps* values range from -0.6 to 0.6 sec/km, *po* values range from - 0.6 to 0.6 sec/km

as the velocity model becomes better determined.

Figure 9 increases the ps aperture to -0.6 to +0.6 sec/km about each plane wave being imaged and the result shows improved spatial resolution as more diffracted energy is included in the final image.

Conclusions

We have shown that modern seismic data can be transformed into source, receiver, or offset plane wave components and these compact data can be imaged to depth with minimal (i.e. source and receiver position independent) travel time computations. Staging over plane wave aperture is a useful tool for velocity analysis as we can concentrate on reflected arrivals and form trial images rapidly. High spatial resolution imaging can be performed by simply adding more source plane wave components as the velocity model becomes better known, which is particularly advantageous for 3D applications. Finally, the methods described here can be implemented for anisotropy by simply changing the vertical delay time algorithm and appropriate amplitude corrections.

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