Spectral-Element FWI: Redefining FWI for complex imaging, an offshore Australia example

Chris Purcell[1], Andrew Woodcock[1], Rich Bartlett[1], Lorenzo Casasanta[1], Sergio Grion[1], Christian Boehm[2], Lion Krischer[2] and Michael Afanasiev[2]

ABSTRACT

The Spectral-Element Method (SEM) offers a high-accuracy alternative to conventional Finite-Difference (FD) approaches for seismic modelling and inversion. FD methods are computationally efficient on quasi-regular Cartesian grids but offer diminished accuracy for complex geological interfaces (e.g., the surface topography) and when dealing with coupled-domain fluid/solid boundary conditions.

In contrast, SEM employs unstructured meshes and high-order polynomial approximations, enabling it to accurately model waveforms in the presence of complex topography, rugose bathymetry and heterogeneous subsurface structures.

With the integration of advanced numerical techniques and GPU acceleration, SEM delivers precise full-waveform modelling and inversion capabilities suited for challenging geophysical scenarios. Field data from offshore Western Australia highlights SEM's ability to accurately discretize intricate seabed geometries and delivers improved results for both waveform modelling and inversion when compared to FD methods.

The following discussion is adapted from Casasanta et al. (2025).

KEYWORDS

FWI, Spectral-Element Method, SEM

INTRODUCTION

The Spectral-Element Method (SEM) is emerging as a high-fidelity alternative to Finite-Difference (FD) methods for addressing challenges in exploration geophysics. While FD methods have long been the cornerstone of seismic waveform modelling due to their computational efficiency on regular grids, FD faced significant limitations in handling complex geometries, material heterogeneities, and advanced boundary conditions.

In contrast, SEM combines the flexibility of unstructured meshes with high-order accuracy, making it particularly well-suited for domains with intricate topography and variable material properties. Recent advancements, such as modular implementations and optimized numerical techniques, have shown SEM's ability to deliver exceptional precision and scalability across diverse applications, including large-scale full-waveform inversion (FWI).

By leveraging SEM, we endeavor to push the boundaries of seismic modelling and inversion to meet the demands of increasingly sophisticated problems in exploration geophysics, as well as emerging applications such as monitoring and high-resolution near-surface characterization. In this work, we review the fundamental principles of SEM, highlight its advantages over FD discretization methods for solving the wave equation, and discuss our implementation strategies that make it viable for geophysical exploration applications relevant to the energy industry. We present examples demonstrating the superior accuracy of SEM using a field dataset from Western Australia. These examples illustrate the advantages of unstructured meshes compared to rectilinear Cartesian grids in both waveform modelling and full inversion applications.

METHOD

The Spectral-Element Method is a higher-order variant of the Finite-Element Method (FEM) that is designed to achieve greater numerical precision in solving the hyperbolic Partial Differential Equations (PDEs) that govern wave propagation in complex media. Seismic waveform modelling is the backbone of computational geophysics, and SEM has the potential to become a transformative tool that offers significant advantages over traditional FD methods for seismic waveform modelling.

SEM leverages higher-order interpolating polynomials and unstructured meshes to achieve superior accuracy and flexibility. Historically, SEM has been primarily employed for global and regional seismic studies (Komatitsch et al., 2002). Recent advancements allow it

[1]Shearwater GeoServices; [2]Mondaic AG

Emails: cpurcell@shearwatergeo.com; awoodcock@shearwatergeo.com; rbartlett@shearwatergeo.com; lcasasanta@shearwatergeo.com; sgrion@shearwatergeo.com; christian.boehm@mondaic.com; lion.krischer@mondaic.com; michael.afanasiev@mondaic.com

to meet the complex demands of exploration geophysics, addressing the need for precise modelling in environments characterized by intricate topography and heterogeneous material properties (Afasaniev et al., 2019).

In our implementation, we leverage one of SEM's most defining features, utilizing unstructured meshes, which provide unmatched flexibility in discretizing complex geological domains. Unlike FD methods, which are confined to structured quasi-regular Cartesian grids, or SEM on cartesian deformed meshes that have been tested with some success in exploration scale geophysics (Trinh et al., 2017), our unstructured approach allows for seamless representation of challenging topographies such as shallow water regions, rugose offshore bathymetry and onshore terrains.

These capabilities make SEM particularly effective for near-surface characterizations in areas where complex interface geometries require accurate implementation of boundary conditions at acoustic, elastic or coupled media interfaces. The weak formulation inherent to SEM ensures the natural continuity of tractions and velocities across fluid-solid boundaries, enabling precise elastic wave propagation modelling in coupled media.

FD methods, by contrast, require computationally expensive grid deformations to approximate these effects (Sethi et al., 2021). This advantage makes SEM an excellent choice for applications such as ocean-bottom node (OBN) acquisition, where accurate wavefield injection and propagation across interfaces are crucial.

SEM supports the direct computation of dynamic quantities such as displacement, particle velocity, strain, and stress wavefield at any point in the computational domain without the need to interpolate the wavefields at the grid point or use FD approximations.

This functionality is particularly valuable for a number of diverse geophysical applications, including emerging applications for CCS monitoring that employ fiber optic distributed acoustic sensing (DAS), ultra-high resolution (UHR) for geotechnical assessments and near-surface characterization for windfarms and geothermal energy sites. By delivering accurate surface and interface wave modelling, SEM could play a central role in energy transition efforts ahead.

PRACTICAL IMPLEMENTATION CONSIDERATION

The integration of SEM using modern software-design principles (Afanasiev et al., 2019) and advanced hardware architecture has been instrumental in making it computationally viable for large scale applications.

The SEM method employs higher-order Lagrange polynomials to represent both wavefields and test functions within each element. By applying Gauss-Lobatto-Legendre (GLL) quadrature at the interpolation nodes, SEM achieves a diagonal mass matrix, eliminating the need for costly matrix inversions during time integration. This key feature not only makes SEM significantly more computationally efficient than traditional Finite-Element methods but also enables highly optimized implementations on modern parallel CPU and GPU architectures. As a result, SEM delivers runtime performance to meet the increasing computational demands of large-scale high-fidelity seismic modelling and inversion.

Meshing for SEM involves discretizing the computational domain into elements that accommodate high-order polynomial basis functions, with the flexibility to use unstructured meshes that conform to complex geometries and material interfaces. The computational cost of SEM simulations scales directly with the number of elements in the discrete mesh, making it crucial to minimize the number of elements while maintaining accuracy.

The ability of our SEM implementation to perform adaptive mesh refinement based on local wavelength plays a vital role in ensuring the practical viability of wavefield simulations. Furthermore, implementing aperture mesh cut-outs, which emulate the finite-difference method's focus on localized shot-gather domains, enables the mapping of the global meshed domain to localized meshes sufficient for computing forward and adjoint modelling for each shot-gather experiment. The inverse mapping of the adjoint simulation back to the global mesh aggregates the full gradient quantities for the given subsurface property, significantly reducing computational overhead while preserving the accuracy of the chosen model parameter updates.

In regions with confined slow velocity layers, such as soft sediments at the water bottom, SEM allows for precise matching of wavelengths. This level of refinement makes high-frequency elastic simulations feasible, which would otherwise be impossible for finite difference methods on quasi-regular FD grids, where slower velocities dictate grid sampling and time-stepping constraints.

Furthermore, local time-stepping (Rietmann et al., 2015) can deliver significant speed-ups for localized low-velocity zones where a relatively small number of fine-size elements would otherwise limit the global time step of the simulation. In the most optimistic scenario, these approaches combined can result in up to an order of magnitude speed-up in both waveform and adjoint modelling while effectively constraining the frequency (f) scaling to the range $fN+1$ and fN, where N is the number of spatial dimensions of the simulation domain.

Advanced non-linear optimization strategies have been seamlessly integrated into our SEM framework, including the Trust-Region Quasi-Newton L-BFGS optimizer and a novel inversion preconditioning technique based on anisotropic diffusion equations. These enhancements significantly improve convergence rates for FWI and enable semi-automated quality control of the inverted subsurface properties.

The anisotropic diffusion preconditioner not only projects the model updates onto the space of geologically plausible models but also aligns naturally with SEM's mathematical formulation and can directly operate on the unstructured meshes. Furthermore, the adoption of a novel numerical formulation for the acoustic wave equation in vertically transverse isotropic (VTI) and tilted transverse isotropic (TTI) media ensures that our SEM implementation remains versatile and effective across a broad range of computational geophysics applications, from acoustic to elastic simulation and inversion.

Together, these innovations establish our SEM implementation as an efficient and scalable tool for geophysical exploration at industry-relevant scales.

FIELD DATA EXAMPLE: MODELLING

We demonstrate the effectiveness of our SEM on unstructured meshes over conventional FD on rectilinear grid modelling and full-waveform inversion examples using a marine streamer field dataset from offshore Western Australia.

The seismic experiment used a dual-source acquisition configuration with 25m flip-flop shooting and a 12.5 m channel spacing. The survey includes 8 cables with 100 m spacing, a maximum offset of 4.6 km, and source/receiver depths of approximately 6–7 m. The survey area covers approximately 450 km^2.

The seabed topography is highly complex, ranging from shallow continental shelf areas (~200 m) to depths exceeding 1200 m, with a network of canyons and submarine mountains. This geological setting provides a benchmark for evaluating the capability of our SEM unstructured meshing to adapt to complex interfaces.

To achieve this, we perform anisotropic acoustic forward modelling at 6 Hz using both FD and SEM methods, and we assess the impact of different model discretization schemes. The FD model employs a rectilinear grid with a spacing of 60 m, corresponding to 4 points-per-wavelength (ppw), which samples the wavefield and model properties.

The FD discretization at the chosen low-frequency and large-wavelength scale, depicted in Figure 1a, exacerbates the stair-casing artifacts (Figure 1c) of the gridded model at the rugose water bottom profile, potentially introducing kinematic or timing errors of up to half a grid cell. In contrast, the SEM approach utilizes our unstructured meshing algorithms, which employ quadrilateral elements of polynomial order 4. These elements are sampled at 1.25 elements-per-wavelength, equivalent to approximately 5 ppw (Figure 1d, blue quadrilaterals). The meshing algorithm not only dynamically adjusts element sizes to match the local wavelength but also deforms and refines the elements to accurately sample the complex seabed topography.

The impact of the discretization on waveform modelling is evident in the common near-channel simulation QC plots (Figures 1e and 1f). The SEM-simulated waveforms demonstrate accurate phase alignment, as indicated by the predominant blue pattern between observed and modelled waveforms. In contrast, the FD-simulated waveforms appear less continuous and energetic, with areas of red patterns caused by phase misalignment due to poor sampling of the complex water-bottom topography.

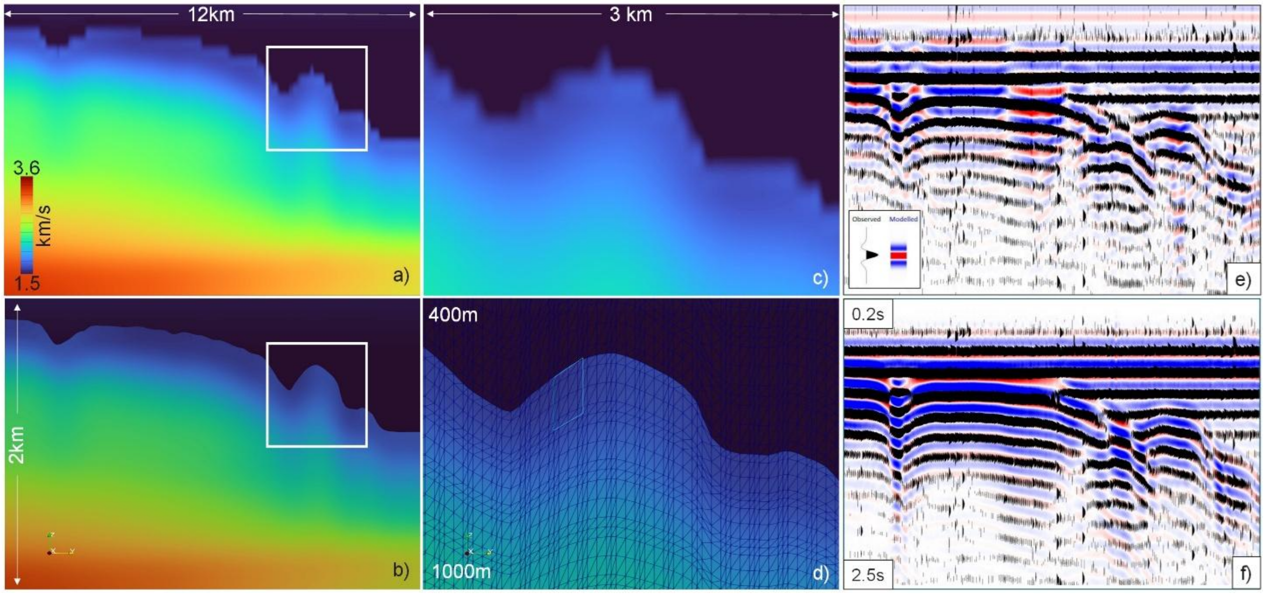

Figure 1: Subsurface velocity models used for FD and SEM forward modelling calculations are shown. Model (a) is discretized using a conventional rectilinear grid with 60 m point spacing, while model (b) is discretized using an unstructured mesh with quadrilateral elements of polynomial order 4. A zoomed-in view (white square) of the seabed topography highlights the FD (c) and SEM (d) models, with the SEM model showing the deformed quadrilateral elements. Near-channel waveforms for FD (e) and SEM (f) are computed at 550 shot locations and compared with observed data. The prominence of blue colors for SEM highlights its improved match to observed data.

FIELD DATA EXAMPLE: INVERSION

We conducted an anisotropic acoustic FWI experiment to update the initial velocity model shown in Figure 2a, using a hierarchical scheme that sweeps through five frequency bandwidth blocks from 5 to 12 Hz. Figures 2b and 2c illustrate the cumulative 12 Hz velocity updates for the FD and SEM methods, which appear broadly similar qualitatively. However, the SEM update demonstrates greater lateral continuity attributed to the anisotropic diffusion preconditioner described earlier.

To evaluate these velocity updates, we performed Kirchhoff depth migration of the recorded seismic data using both the initial model (Figure 2d) and the final FD and SEM models (Figures 2e and 2f). For comparison, the migrated seismic sections are overlaid with the respective velocity models. The white dotted rectangle highlights a region located beneath a canyon in the seabed topography, where SEM imaging is deemed superior to FD imaging as confirmed by the improved gather flatness in the area (Figure 3).

We hypothesize that this improvement is due to the enhanced bathymetry discretization provided by the SEM method, which reduces modelling errors, improves data misfit and, consequently, betters the quality of the FWI velocity updates. This represents one of the first successful benchmarks of this kind in the seismic imaging industry.

CONCLUSIONS

We demonstrate the improved accuracy of our SEM framework using field data from offshore Western Australia. The results highlight the advantages of SEM on unstructured meshes over traditional FD methods on rectilinear Cartesian grids, in both full-waveform modelling and inversion applications.

SEM proves highly effective for wavefield simulation due to its precise handling of complex interfaces and boundary conditions, for both free-surface and fluid-solid interfaces. Overall, SEM represents an advancement in computational geophysics, offering geometric flexibility, accuracy and computational efficiency.

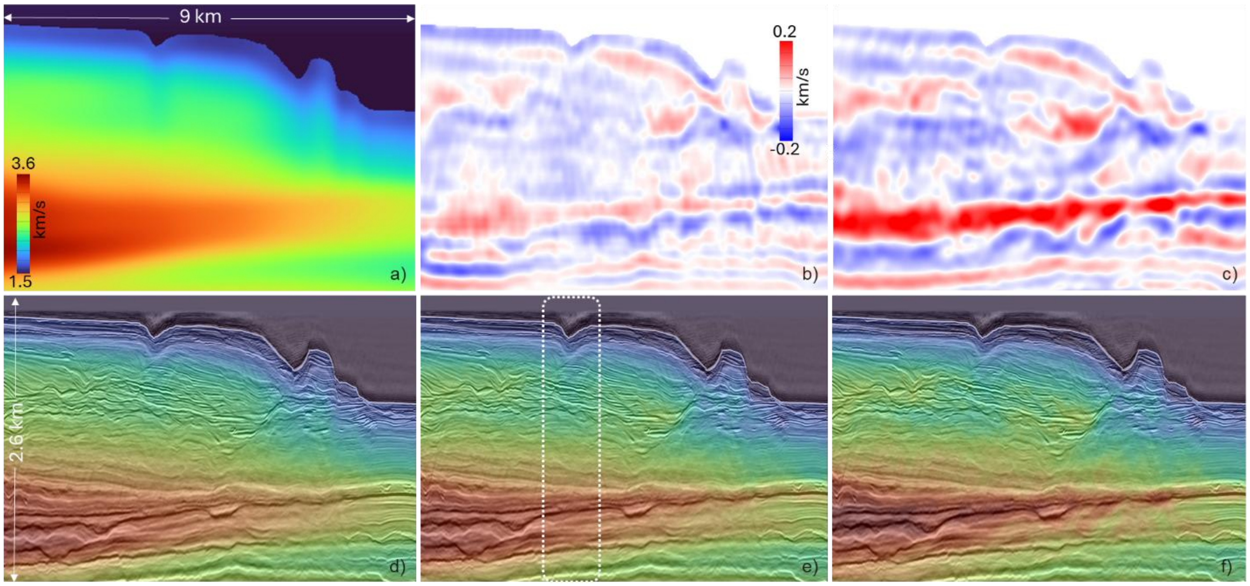

Figure 2: (a) Initial velocity model for the 5–12 Hz full-waveform inversion. Panels (b) and (c) show the cumulative velocity updates (velocity difference, dV) for FD and SEM, respectively. Panel (d) displays the initial velocity model overlaid with the Kirchhoff depth-migrated seismic data, while panels (e) and (f) present the final FD and SEM velocity models, respectively, with migrated seismic as the background. The white dotted square highlights a region where SEM imaging is considered superior to FD imaging.

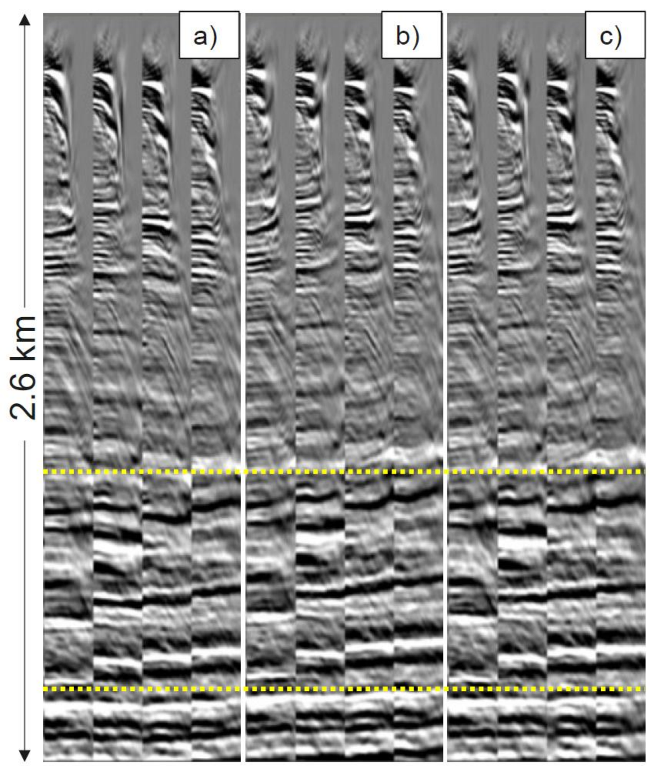

Figure 3: Kirchhoff Depth Migration Offset Gathers obtained with the initial (a) and final FD (b) and SEM (c) velocity models from Figure 2. The gathers are from the area highlighted by the white squared dotted rectangle. The maximum imaged offset is 3 km.

By addressing the limitations of FD methods and leveraging modern computational resources, our SEM implementation will be the foundation technology for tackling the increasingly complex demands of exploration geophysics and for applications beyond geophysics. As innovations in hardware and numerical techniques continue to evolve, SEM's role in advancing seismic modelling and inversion will only grow stronger, opening new possibilities in geophysical research and industrial applications.

ACKNOWLEDGEMENTS

The authors would like to thank Shearwater GeoServices and Mondaic AG for their support and permission to publish this work. The authors are also indebted to Geoscience Australia for providing the field dataset used for the examples.

REFERENCES

Afanasiev, M., C. Boehm, M. van Driel, L. Krischer, M. Rietmann, D. A. May, M. G. Knepley, and A. Fichtner, 2019, Modular and flexible spectral element waveform modelling in two and three dimensions, Geophys. J. Int., **216**(3), 1675-1692. https://doi.org/10.1093/gji/ggy469

Casasanta, L., C. Purcell, S. Grion, C. Boehm, L. Krischer, M. Afanasiev, 2025, Spectral-Element FWI: A high-fidelity alternative to finite-difference for complex geophysical challenges, 86th Annual International Conference and Exhibition, EAGE, Extended Abstracts, 1-5. https://doi.org/10.3997/2214-4609.202510770

Komatitsch, D., and J. Tromp, 2002, Spectral-element simulations of global seismic wave propagation, part II: 3-D models, oceans, rotation, and gravity, Geophys. J. Int., **149**(2), 390-412. https://doi.org/10.1046/j.1365-246X.2002.01653.x

Rietmann, M., D. Peter, O. Schenk, B. Ucar, and M. Grote, 2015, Load balanced local time stepping for large-scale wave propagation, IEEE Int. Parallel Distrib. Process. Symp., 925–935. https://doi.org/10.1109/IPDPS.2015.10

Sethi, H., J. Shragge, and I. Tsvankin, 2021, Mimetic finite-difference coupled-domain solver for anisotropic media, Geophysics, **86**(1), T45-T59. https://doi.org/10.1190/geo2020-0092.1

Trinh, P.-T., R. Brossier, L. Metivier, L. Tavard, J. Virieux, and P. Wellington, 2017, Efficient 3D elastic FWI using a spectral-element method on Cartesian-based mesh, 87th Annual International Meeting, SEG, Expanded Abstracts, 1533-1538. https://doi.org/10.1190/segam2017-17587770.1

BIOGRAPHIES

Chris Purcell graduated from University of Leicester (UK) in 2000 with a master's degree in geophysics. He then joined Veritas DGC (/CGG) as a geophysicist where he spent 20 years working mainly on velocity model building and imaging projects. Chris joined Shearwater GeoServices in 2020 and is currently Senior Advisor for Imaging in Shearwaters' Global Technical Group.

In this role he delivers technical advice, hands on support and training to Shearwater's geophysicists across global centres. He works closely with R&D teams and Reveal Software support teams in client support, developing, testing and rolling out of new model building and imaging technology, including Shearwater's Spectral Element FWI platform.

Andrew Woodcock graduated from the University of Leeds (UK) with a master's degree in exploration geophysics. From 2003 to 2016, he worked as a seismic processor at Veritas DGC (/CGG), initially offshore, before focusing primarily on depth imaging projects in the UK office. He joined Shearwater GeoServices in 2017, where he currently holds the position of Imaging Advisor.

Andrew has been involved in many technically demanding projects, ranging from single-process-focused testing to full project management. He actively collaborates with geoscientists, clients, and R&D teams to help deliver the best technology effectively and on time. Additionally, Andrew contributes to the delivery of technical training programs and provides support to both internal users and external Reveal users.

Rich Bartlett graduated in 2002 with a masters in geophysics jointly awarded by the University of Leeds (UK) and Queen's University (Canada). He spent his early career as a geophysicist at Veritas DGC (/CGG/CGGVeritas) undertaking various roles in Depth Imaging teams in London and Calgary. This was followed by a period of consulting with Monarch Geophysical Services. He moved to Dolphin Geophysical in 2013, now Shearwater GeoServices, progressing from Depth Imaging Manager to UK Processing Manager. He accepted his latest challenge in 2023 taking on the role of Global Technical Manager for Processing and Imaging.

Rich has helped build Shearwater's processing capabilities from its foundations, supported its Reveal software development and the technical standards/procedures by which Shearwater teams operate. Shearwater offers a wide range of processing, imaging and inversion services behind its vessels, and to the proprietary and multiclient seismic markets.

Lorenzo Casasanta is Principal Researcher and Head of Imaging and HPC R&D at Shearwater GeoServices, where he leads advanced research in seismic imaging, full-waveform inversion, and high-performance computing. His work spans algorithm development, large-scale GPU/CPU optimization, and collaborative projects with industry and academic partners. Lorenzo has a background in telecommunication engineering (M.Sc./Ph.D., Politecnico di Milano, Italy) and over 15 years of experience advancing geophysical imaging technologies in both commercial and research environments.

Dr. Sergio Grion is currently Chief Geophysicist - Processing Technology at Shearwater GeoServices, leading research on seismic data processing and imaging methods and advising the company on research directions and adoption of new technologies. After completing his M.Sc. in electronic engineering at the Politecnico of Milan (Italy) in 1995 and a Ph.D. in Earth Sciences at the University of Milan in 1999, he joined WesternGeco, where he worked as R&D geophysicist until 2004. From 2004 to 2012 he was with CGG, ultimately as Senior Advisor and Head of multicomponent R&D. He is with Shearwater since 2013. Dr. Sergio Grion is a member of EAGE, SEG and GESGB and is author of more than 100 technical papers and articles. His most cited work is on Mirror imaging of OBS data. He regularly serves as committee member and chairman during annual conferences and workshops. He is the recipient of the 1998 EAGE Loránd Eötvös award.

Christian Boehm received his Ph.D. in applied mathematics from the Technical University of Munich (Germany) in 2015. He subsequently held Postdoctoral and Senior Researcher positions at the Institute of Geophysics, Department of Earth Sciences, ETH Zurich (Switzerland). His research focuses on large-scale computational inverse problems, with a particular emphasis on full-waveform inversion.

In 2018, he co-founded Mondaic, an ETH Zurich spin-off that develops software for digital twin modelling, waveform simulations, and quantitative imaging. Mondaic's technology is applied across a broad range of scientific and industrial domains, including geophysics, civil engineering, and biomedical engineering. From 2023 to 2025, he served as Conference Chair for the Ultrasound Tomography and Imaging track at SPIE Medical Imaging.

Lion Krischer earned a doctoral degree from the Ludwig-Maximilian University of Munich (Germany) in 2017 working on scaling full waveform inversions to large data volumes. His work sits at the crossroads where seismics meets computational science, Big Data engineering, and machine learning. A major focus of his has always been the development and maintenance of production-grade scientific software using agile techniques.

For over a decade he has been a core developer of several open-source and commercial software packages pivoting around data orchestration, signal processing, cloud computing, and waveform simulations with a world-wide user base. He co-founded Mondaic in 2018 - an ETH Zurich spinoff creating software for digital twins, waveform simulations, and quantitative imaging - used widely in geophysical studies, civil-infrastructure projects, and non-destructive testing applications.

Michael Afanasiev obtained his Ph.D. in Geophysics from ETH Zurich (Switzerland) in May 2017 for work related to multi-scale full waveform inversion. He has a bachelor's degree in computational physics and a master's degree in geophysics from McMaster University and Western University respectively (both located in Canada). He is a core developer of Salvus and has won several awards for his work on full-waveform inversion from the CSEG, EAGE, and AGU societies.