Seismic amplitude versus angle modeling of a bottom-simulating reflector

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Summary

Gas-hydrates have attracted the scientific community because of their widespread occurrences, potential as future energy resources and role in environmental hazard. Presence of gas-hydrates can be established through seismic experiment by mapping a reflector, known as bottom simulating reflector or BSR, which represents the base of gas-hydrates-laden sediment underlain by free-gas or brine saturated sediments. The amplitude variation with angle (AVA) responses from BSR contains useful information for quantitative assessment of gas-hydrates. To examine the suitability of the AVA function for linearised inversion, we perform a test based on Residual Function Mapping (RFM), which shows that all parameters except S-wave velocity pairs are well invertible. The decreasing size of contours with increasing angle or larger offset data indicates that the larger the maximum angle or offset is, the better the AVA inversion. The AVA modeling also helps to understand the nature of BSR.
Key words: Gas-hydrates, BSR, AVA, RFM test, AVO inversion.

Introduction

Gas-hydrate is a crystalline substance of water and light hydrocarbons (mainly methane) that forms at high pressure and low temperature, and is found in two distinct regions of (i) permafrost and (ii) outer continental margins (Sloan 1990; Kvenvolden 1998). It has attracted the global attention because of its widespread occurrences, potential as future energy resource, possible role in climate change, influence on submarine geo-hazard, and relationship to fluid flow in accretionary wedges. Gas-hydrates can trap free-gas underneath. Therefore, quantification of gas-hydrates is very essential for evaluating the resource potential and hazard assessment.

Pure hydrates have much higher seismic velocities than those of sediments in which they occur. Hence presence of hydrates increases the seismic velocities, whereas even a small amount of free-gas below the hydrates-bearing sediments causes a considerable decrease in seismic velocity. Thus the BSR, representing the boundary between gas-hydrates-bearing sediments above and free-gas saturated sediments below, acts as a very good seismic reflector that can be identified by a seismic experiment. The BSR may also be caused by high concentration of gas-hydrates underlain by brine saturated sediments. The amplitudes versus angle information can be used to determine various elastic parameters like the P-wave velocity (V_P), S-wave velocity (V_S) and density (ρ) of both hydrates and free-gas-bearing sediments, and hence provide useful information for quantification of gas-hydrates and/or free-gas across a BSR.

Here we compute the AVA responses for several gas-hydrates models and study their characteristic behavior. The AVA is mainly governed by Zoeppritz equation (Zoeppritz, 1919). However, the mathematical expression of the equation is complex in nature and making it difficult to distinguish how amplitude varies with slight change in rock property. To give an intuitive understanding of how these amplitudes are related to various physical parameters, a number of approximations to the Zoeppritz equation have been made by several scientists (Aki and Richards 1980; Shuey 1985), which are valid for small angle of incidence and low impedance contrast. Since, the Zoeppritz equation is a non-linear function of model parameters, the approximation to the exact equation causes an error in determining the model parameters. Therefore, to exploit the full information contained in the AVA data, we need to use the exact equation. Prior to performing any inversion of AVA data to estimate elastic parameters across a BSR, we carry out the Residual Function Mapping (RFM) test (Demirbag and Costain, 1993) with a view to find out the invertible elastic parameters and to examine the role of higher angle or offsets towards the convergence.

Theory

When a seismic ray strikes a plane boundary at non-zero angle, partition of energy takes place. Figure 1 illustrates the reflection, transmission and mode conversion of a plane P-wave at a boundary separated by three elastic parameters (V_P, V_S, and ρ) on either side. Investigation of angle or offset-dependent reflectivity guided by Zoeppritz equations (Zoeppritz 1919) has started long back (Ostrander 1984; Castagna et al. 1993). This dictates the determination of reflection and transmission coefficients or amplitudes for both P- and converted S-waves as a function of angle. For easy numerical solution, the Zoeppritz equations can be written through a matrix form (Waters 1977) as

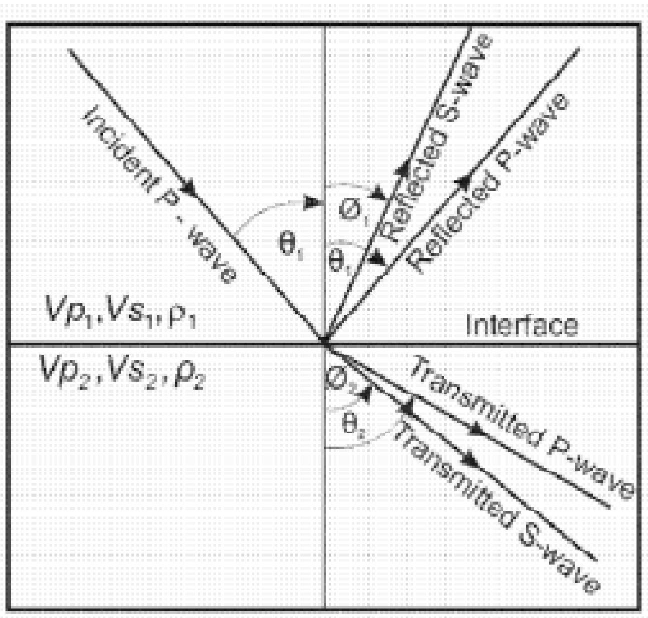

Fig. 1. Ray diagram for the partition of plane wave energy at a horizontal interface.

$$\mathbf{Q}=\mathbf{P}^{-1}\mathbf{R} \qquad (1)$$

Where **P** is a 4 × 4 square matrix and, **R** and **Q** are 4 × 1 column matrices shown below.

$$\mathbf{P}=\begin{bmatrix} \sin\theta_1 & \cos\varphi_1 & -\sin\theta_2 & \cos\varphi_2 \\ -\cos\theta_1 & \sin\varphi_1 & -\cos\theta_2 & -\sin\varphi_2 \\ \sin 2\theta_1 & \frac{V_{P1}}{V_{S1}}\cos 2\varphi_1 & \frac{\rho_2 V_{S2}^2}{\rho_1 V_{S1}^2} 2\sin\theta_1\cos\theta_2 & -\frac{\rho_2 V_{P1} V_{S2}}{\rho_1 V_{S1}^2}\cos 2\varphi_2 \\ \cos 2\varphi_1 & -\frac{V_{S1}}{V_{P1}}\sin 2\varphi_1 & -\frac{\rho_2 V_{P2}}{\rho_1 V_{P1}}\cos 2\varphi_2 & -\frac{\rho_2 V_{S2}}{\rho_1 V_{P1}}\sin 2\varphi_2 \end{bmatrix}$$

$$\mathbf{Q}= [R_{PPR} \quad R_{PSVR} \quad R_{PPT} \quad R_{PSVT}]^{T} \text{ and}$$

$$\mathbf{R}= [-\sin\theta_1 \ -\cos\theta_1 \ \sin 2\theta_1 \ -\cos 2\phi_1]^{T}$$

where R_{PPR} and R_{PSVR} are the reflection coefficients and, R_{PPT} and R_{PSVT} are transmission coefficients of P-P and converted P-S waves respectively. Angles (θ and ϕ) are defined in Figure 1.

Seismic velocities in hydrates and gas-saturated sediments.

As gas-hydrates have much higher seismic velocity compared to that of pore-fluids, hydrates-bearing sediments show higher velocity than that of normal oceanic sediments in which they normally occur. Accurate estimation of elastic parameters is very important to translate the velocity build-up in terms of concentration of gas-hydrates. There are a number of rock physics models proposed by different scientists (Lee et al. 1996; Ecker et al. 1998; Helgerud et al. 1999; Jacobsen et al. 2000) to calculate the seismic velocities of hydrates-bearing sediments. In many models, gas-hydrates have been considered as a part of pore fluid without any porosity reduction. Since gas-hydrates are solid, we consider the hydrates as a part of the rock-frame and accumulated in the pore spaces with porosity reduction. Gas-hydrates make the porous sediments impervious and hence hydrated sediments trap free-gas underneath. Even a small amount of free-gas below the BSR reduces the seismic velocity considerably and has thus remarkable effect on AVA.

Here, we calculate the P- and S-wave seismic velocities for gas-hydrates and gas bearing sediments based on the Biot-Gassmann theory modified by Lee (Lee 2002, 2004), termed as BGTL using the effective pressure, $P=(\rho-\rho_w)sz$ (Helgerud et al. 1999), where ρ is the sediment bulk density; ρ_w is water density; g is the acceleration due to gravity and z is the depth below seafloor. The variation of P- and S-wave seismic velocity with saturation of gas-hydrates and free-gas are shown in Figure 2. The various constants used in calculating the seismic velocities are shown in Table 1.

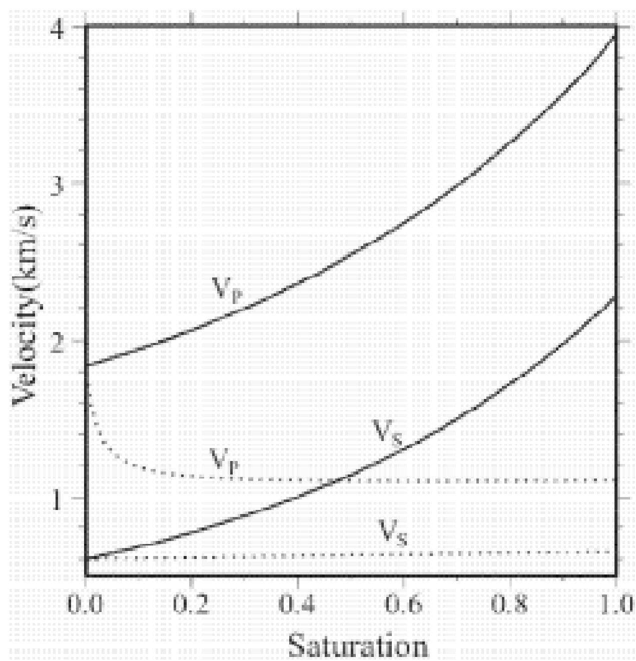

Fig. 2. Variation of P-wave (V_P) and S-wave (V_S) seismic velocities with gas-hydrates (solid lines) and gas (dotted lines) saturation respectively.

Table 1: *Parameters used for theoretical calculation*

Component	K(GPa)	G(GPa)	ρ (kg/cc)
Quartz	36	45	2650
Clay	20.9	6.85	2580
Hydrate	6.41	2.54	910
Methane *	.033	-	112
Water	2.25	-	1000

* *Taken from Dvorkin et al. (1999)*

RFM test

We perform the RFM test using the Zoeppritz equation to find out the invertible elastic parameters and examine the effect of maximum offset or angle towards the convergence. The error E (P1, P2) in RFM is defined as

$$E(P_1, P_2) = \sum_{i=1}^{N} \left[R_{PPRi} - R*_{PPRi}(P_1, P_2)\right]^2 \qquad (3)$$

Where R_{PPRi} is the AVA response for a given set of elastic parameters and $R*_{PPRi}(P_1, P_2)$ is the response by changing the values of two parameters, keeping values of all other parameters to their true values at the ith observation. N is the total number of observation points. The parameters used for RFM tests are shown in Table 2.

Table 2: *Various model parameters across a BSR used for* RFM test

Layer	V_P (m/s)	V_S (m/s)	ρ (kg/m^3)
Hydrate-bearing sediment (40% hydrate saturation)	2400	1700	1800
Gas-bearing sediment (10% gas saturation)	1200	600	1800

AVA Modeling

We have computed the AVA responses from a BSR using the Zoeppritz equation. The computed responses for various fixed saturation of free-gas with different concentration of gas-hydrates are shown in Figure 3. Following are some important observations made from the AVA modeling.

In presence of free-gas, the negative RC increases monotonically with increasing angle for a fixed gas-hydrates concentration upto ≤30% (Figure 3). For >30% gas-hydrates saturation, the negative RC decreases initially and then starts increasing with angle irrespective of underlying free-gas saturation. This AVA behavior is similar to the case of 0% free-gas irrespective of overlying gas-hydrates concentration.

We have three parameters (Vp, Vs and ρ) on either side of an interface. For investigating the degree of linearity in non-linear function (Zoeppritz equation) and hence the feasibility of performing the AVA inversion, we carry out the RFM test as per equation (3). Closed contours with a single minimum and/or smaller in contour size are the guidelines to understand the degree of linearity between two parameters. Since densities appear in the form of ratio in Zoeppritz equation, the ratio is considered as a single parameter. As densities due to presence of hydrates and free-gas are almost unchanged, the density ratio can be considered as 1 across the BSR. Therefore, we get 6 pairs of model parameters in performing the RFM test. To show the effect of maximum

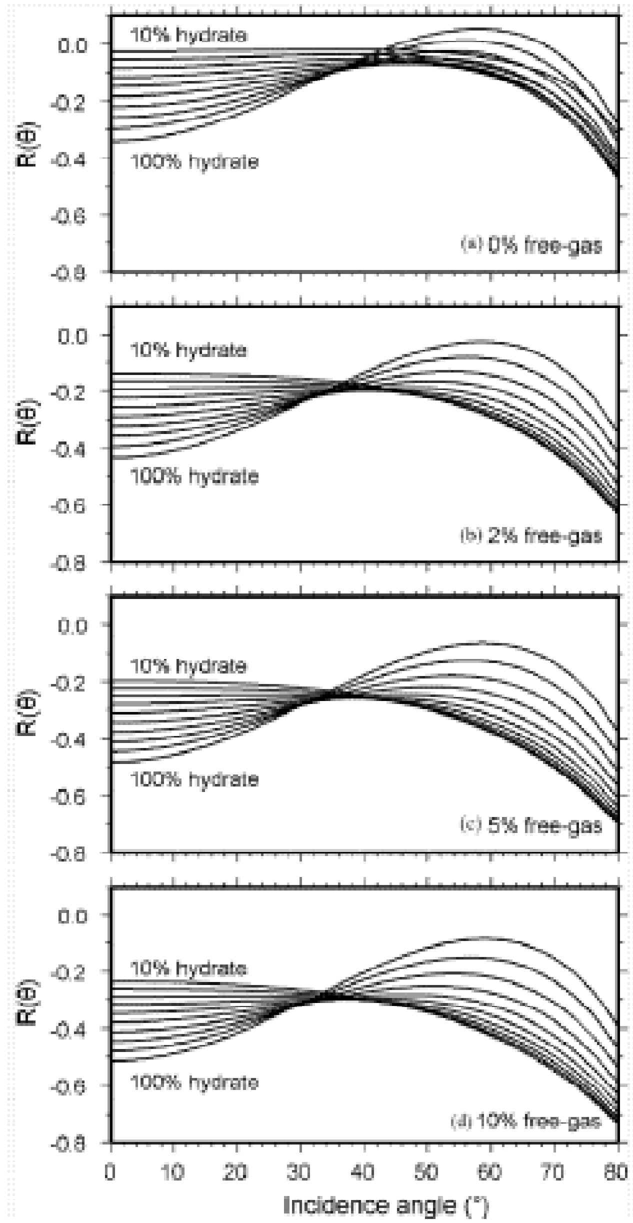

Fig. 3. AVA responses from BSR for different concentration of gas-hydrates at (a) 0%, (b) 2%, (c) 5% and (d) 10% saturation of free-gas.

angles, first we calculate the responses at interval of 1° upto a maximum of 30° and then up to 40° starting from 0°. The results of RFM test around a large neighborhood of chosen model parameters (Table 2) are displayed in Figure 4. All combinations except for the V_{S1} and V_{S2} pair show good degree of linearity. We also see that the size of contours decreases with increasing angle of data. This implies that the degree of linearity becomes better and better with the increase in maximum angle of incidence. As the S-wave velocity pair displays poorly defined minima irrespective of maximum angle, ambiguous result may be obtained if AVA inversion is performed for the S-wave velocity pairs.

It is difficult to determine the S-wave velocities across the BSR simultaneously. The convergence of S-wave velocity with any other elastic parameter shows that one S-wave velocity can be inverted. For marine seismic AVA data, since we know the seismic velocities and density of water, we can invert velocities and density of the seafloor sediments. By holding fixed the above three parameters as top parameters, we can determine the bottom three parameters associated any reflector below the seafloor. Thus in a layer stripping fashion, we can determine all elastic parameters associated with successive deeper layers. Since the BSR is a physical property interface, not a lithological boundary, the layer stripping approach has to be modified slightly while modeling the AVA data from the BSR. The top of the gas-hydrates saturated layer is gradational in nature and difficult to identify on seismic data. The concentration of gas-hydrates and hence V_P and V_S increase with depth. The seismic velocities become maximum at the BSR. Presence of free-gas below the BSR decreases the V_P considerably but does not affect the V_S. So the V_S below the BSR can be considered as that derived for the bottom layer of the reflector just above the BSR. The density of gas-hydrates-bearing sediments and the free-gas saturated sediments across the BSR remains almost the same as that of the background density.

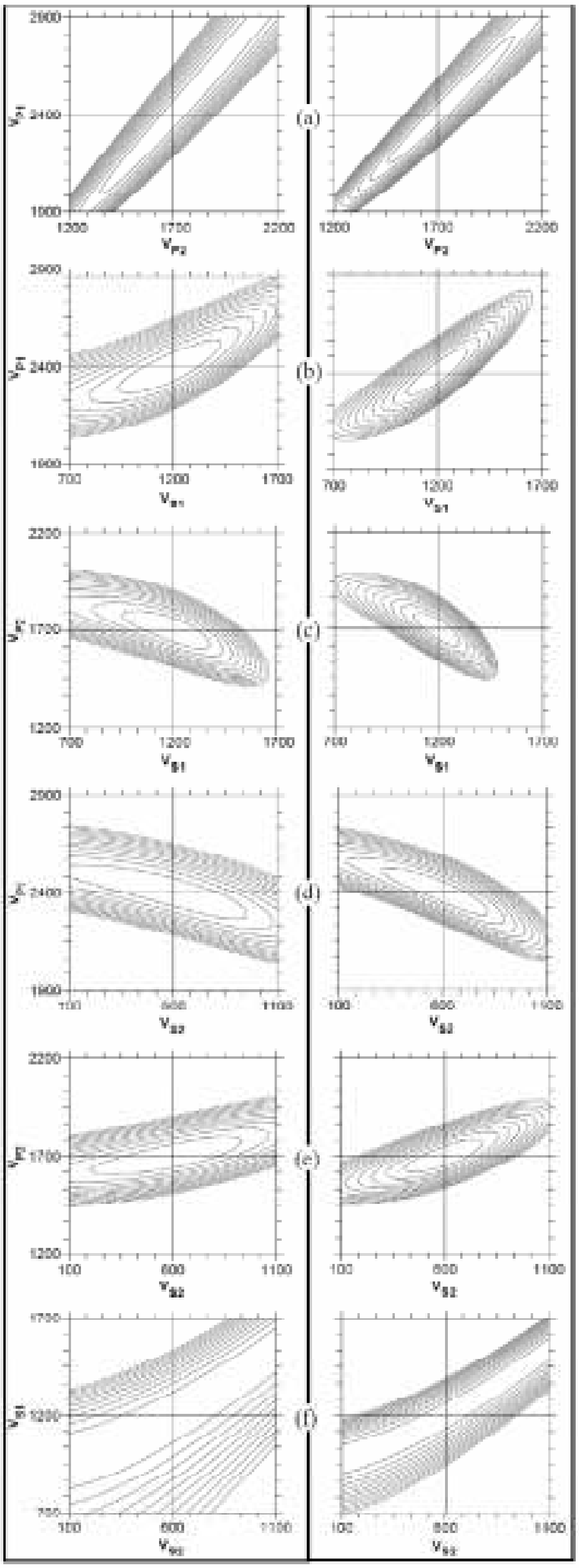

Fig. 4. Residual function mapping for AVA responses upto maximum angle of 30º (laft panel) and 40º (right panel), investigating the linear dependence between (a) V_{P1} and V_{P2}, (b) V_{P1} and V_{S1}, (c) V_{P2} and V_{S1}, (d) V_{P1} and V_{S2} and (e) V_{P2} and V_{S2} and (f) V_{S1} and V_{S2} pairs. Crossing hair marks true values. Contour values range from 0.01 to 0.1 in steps of 0.01. Velocities are in m/s.

Conclusions

In presence of free-gas, the reflection coefficient increases with incident angle and hence free-gas can be identified if the saturation of gas-hydrates is less than 30%. In absence of free-gas, the negative reflection coefficient decreases at first and after that increases with angle irrespective of gas-hydrates concentration. This behavior is similar to the case of >30% gas-hydrates concentration irrespective of free-gas saturation.

The results of RFM test with different pairs of model parameters show closed contours with single minima for most of the pairs, except for the S-wave velocity pair. This indicates that the respective model parameters are invertible by inversion of AVA data except for the S-wave velocity pair. We also see that the size of contours decreases with increase in maximum angle or offset. This implies that the larger the maximum angle or offset is, the better the degree of linearity in non-linear AVA inversion is.

The layer stripping approach can be used to determine the elastic parameters associated with various reflectors including the BSR.

Acknowledgements

We are grateful to the Director, NGRI for his kind permission to publish this work. CSIR, New Delhi is gratefully acknowledged for financial support through SRF to M. Ojha.

References

Aki, K., and Richards, P. G. (1980) Quantitative seismology:

theory and methods (Eds.), W. H. Freeman and co., New York.

Castagna, J. P., Batzle, M. L., and Kan, T. K. (1993) Rock physics: the link between rock properties and AVO response, In: Castagna, J. P., Backus, M. M. (Eds.), Offset-dependent reflectivity-theory and practice of AVO analysis. Society of Exploration Geophysics, 135-171.

Demirbag, E., Çoruh, C., and Costain, J. C. (1993) Inversion of P-wave AVO, In: Castagna, J. P., and Backus, M. M. (Eds.), Offset-dependent reflectivity - Theory and Practice of AVO Analysis. Society of Exploration Geophysics, 287-302.

Dvorkin, J., Moos, D., Packwood, J.L. and Nur, A. (1999) Identifying patchy saturation from well logs. Geophysics, 64, 1759-1759.

Ecker, C., Dvorkin, J. and Nur, A. (1998) Sediments with gas hydrates: Internal structure from seismic AVO. Geophysics, 63, 1659-1669.

Jacobsen, M., Hudson, J.A., Minshull, T.A. and Singh, S.C. (2000) Elastic properties of hydrate-bearing sediments using effective medium theory. Journal of Geophysical Research, 105, 561-577.

Helgerud, M. B., Dvorkin, J., Nur, A., Sakai, A. and Collett, T. (1999) Elastic-wave velocity in marine sediments with gas hydrates: effective medium modeling. Geophysical Research Letters, 26, 2021-2024.

Kvenvolden, K.A. (1998) A primer on the geological occurrence of gas hydrate. In Henriet, J. P. and Mienert, J. (editors), Gas hydrates: Relevance to the World Margin Stability and Climate Change. Geological Society, London, Special Publications, 137, 9-30.

Lee, M. W., Hutchinson, D. R., Collett, T. S. and Dillon, W. P. (1996) Seismic velocities for hydrate-bearing sediments using weighted equation. Journal of Geophysical Research, 101, 20347-20358.

Lee, M.W. (2002) Biot-Gassmann theory for velocities of gas-hydrate bearing sediments. Geophysics, 67, 1711-1719.

Lee, M. W. (2004) Elastic velocities of partially gas-saturated unconsolidated sediments. Marine and Petroleum Geology, 21, 641-650.

Ostrander, W. J. (1984) Plane-wave reflection coefficients for gas sands at non-normal angles of incidence. Geophysics, 49, 1637-1649.

Shuey, R. T. (1985) A simplification of Zoeppritz equations. Geophysics, 50, 609-614.

Sloan, E. D. (1990) Clathrate hydrate of natural gases. Marcel Dekker, Inc. (Ed.).

Waters, K. H. (1997) Reflection Seismology. A wiley interscience publication, California (Ed.), V1, pp6-42.

Zoeppritz, K. (1919) On the reflection and penetration of seismic waves through unstable layers. Erdbebenwellen Viii B, (Goettinger Nachr, Ed.), Pp. 66-84.