9th Biennial International Conference
& Exposition on Petroleum Geophysics

"HYDERABAD 2012"

P-017

Direct Current VES Data Inversion using Singular Value Decomposition Method for Delineating Seawater Intrusion in parts of Konkan, Western Maharashtra

*Saumen Maiti[1], Vinit C Erram[1], Gautam Gupta[1], Rahul Nandi[2] and Santanu Pal[2]

Summary

Singular value decomposition (SVD) based geophysical inversion method is developed and successfully applied to the modeling of resistivity profiles from the hard rock area of Konkan, coastal Maharashtra. 58-number of vertical electrical sounding (VES) collected over Kudal-Vengurla- Malvan and its surrounding area is modeled using the new methods. The results are interpreted in light of the geological setting to know the aquifer infected with the saline water in the study area. Prior to applying the method on actual resistivity data, the efficiency of the method was tested on simulating the synthetic signals generated possible combination of resistivity and layer thickness for a large number of hypothetical setting. The inversion results based on the new algorithm suggest a very high resistivity structure in the north-eastern part of the area, which could be caused due to the presence of laterites. In the south- eastern part of the study area, a very high resistivity zone is observed, which is also caused possibly due to the presence of laterites. The NE-SW trending major lineaments and its criss-crosses are indentified from the apparent and true resistivity surface map. In the south-western part, a very high conductive zone is observed near the coast indicating extensive influence of saltwater water intrusion. Our results also show that the effect of intrusion of saline water diminishes from the southern western part to the north eastern part of the region. Two dimensional modeling of four resistivity profiles data from the study region shows that the flow of saline water from the coastal side partly controlled by the lineaments and conduits. The method is robust for modeling resistivity data in hard rock terrain and the results obtained here are useful for interpreting the fractures, major joints and lineaments, and also for constraining the geophysical models used for the study of the drainage pattern in the hard rock area.

Introduction

Groundwater is a major source of drinking and domestic needs and therefore, its accessibility assumes a great significance for the society. In the coastal area of Maharashtra, which is in contact with the Arabian Sea, groundwater level fluctuates in response to tidal variation (Todd, 1980). The pollution of groundwater by seawater occurs when saline water displaces or mixes with freshwater (Song, et al, 2007). Consequently, the layered structures in and around the region have influenced the near surface distribution pattern of electrical properties. Hence, the modeling and interpretation of Direct Current (DC) resistivity sounding in this region assume a special significance to understand the inhomogeneous infiltrations of fluids through pores and geologically weak zones, such as faults and fractured zones, fluid percolation pattern near the sub-surface area and the sea water intrusions studies.

[1]Indian Institute of Geomagnetism, NewPanvel-410218, India, [2]Indian School of Mines, Dhanbad-826004, India
E-mail:saumen_maiti2002@yahoo.co.in

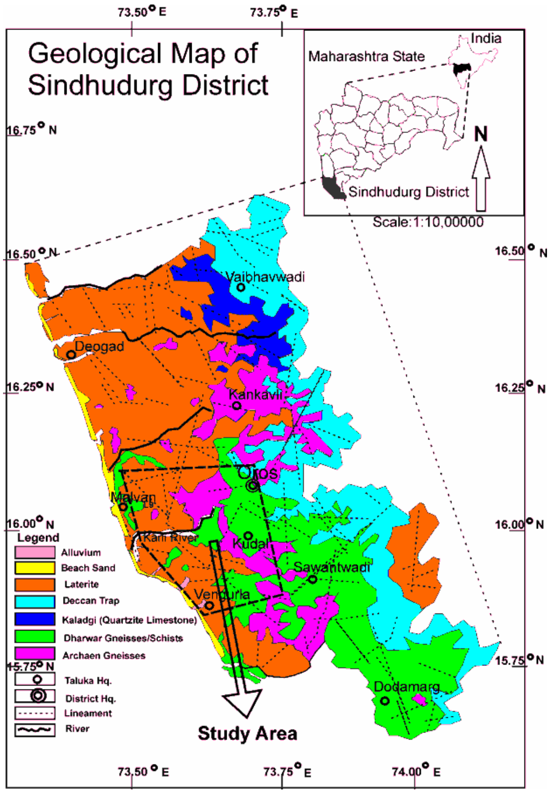

Figure 1: Geological map of the study area.

One of the most common methods for assessing seawater intrusion through an aquifer in coastal area is a periodic analysis of ground water chemistry. When seawater intrusion is main cause of high salinity, ground water generally exhibits high concentration not only in total dissolved solids (TDS) but also in major cautions and ions (Todd, 1980). In general with increase of distance from coastline, amplitude of fluctuation decreases and time lag increases. To analyze the variation of ground water level and its quality due to seawater intrusion and tidal variation, periodic measurement of ground water level and analysis of quality are necessary, which require installation of monitoring well. However, appropriate installation of well would be difficult without knowing the extent of seawater intrusion a priori. Thus, surface geophysical methods such as electrical resistivity surveys or electromagnetic method could be a best alternative for this purpose.

Direct Current (DC) resistivity sounding method is one of the most popular methods that have been extensively applied for solving hydrological, geothermal, environmental and engineering problems (Zohdy, 1989; Ekinci, and Demirci, 2008). In the DC resistivity method, current is introduced directly into the ground through a pair of current electrodes and resulting voltage difference is measured between a pair of potential electrodes. The method provides the apparent resistivity distribution against depth. The depth of penetration of electrical signal is generally found to be approximately one-third of the distance between the electrode separations. The present study area covers major parts of Sindhudurg district of Maharashtra.

The Sindhudurg district of Maharashtra, which is covered by the Deccan volcanic rocks, most of the soils are derived from lateritic rocks and the groundwater, is circulated through a network of voids, conduits, joints and fractures. Hence monitoring the shallow distribution of true resistivity pattern in the area is vital for mapping the faults, fractures, joints, conduits and lineaments for groundwater exploration. Forward mathematical models are generally used to relate the measured/observed data (here apparent resistivity) to desired model parameters (true resistivity and layer thickness). The forward modeling is the process of predicting results of measurements on the basis of some general principle or model and specific conditions relevant to the problem at hand. However, inferring the true resistivity distribution from the apparent resistivity data does not provide precise information due to inherent nonlinearity in the data structures.

The relation between the observed "apparent resistivity" and model parameter ("true resistivity" and "layer thickness") is non-linear. Hence the estimation of true resistivity distribution against depth from apparent resistivity data essentially leads to solving the inverse problem. Inverse modeling (or "inversion") in contrast starts with data and a general principle or model, estimates model parameters by minimizing the error/misfit function set up between the data and model parameter (Menke, 1984). Several attempts have been made to solve the resistivity inverse problems (Ghosh 1971; Zohdy, 1989; Macias et al., 2000; Qady, and Ushijima, 2001; Ekinci, and Demirci, 2008). However, these approaches have some common drawbacks. These algorithms critically depend on the initial parameter chosen for it. The general class "Monte Carlo" e.g., genetic algorithm, simulated annealing is proven to be useful whilst a good staring model is not available (Rubinstein, 1981; Kirkpatrick 1983) for offering global solution. However, these methods are computationally expensive. Sometimes it is unfeasible for processing voluminous data. Thus it is imperative to search for more robust cost effective approach for solving non-linear resistivity inverse problem.

In the present work, a singular value decomposition (SVD) based inversion scheme is used for estimating true resistivity and layer thickness precisely from apparent resistivity data collected over an area in grid form of area covering (58 number of Schlumberger sounding data) of Kudal-Vengurla region (Latitude 15.85° N to Latitude 16.15°N) to (Longitude 73.53° E to Longitude 73.8° E) Maharashtra (Figure 1 and 2). Thus, a SVD based scheme is introduced for solving non-linear inverse problem which brought out precise distribution of true resistivity which is of importance especially the issues related to groundwater exploration in Kudal-Vengurla, Konkan region.

Theory and /or Method

Schlumberger sounding is the most popular sounding method in DC resistivity survey. For Schlumberger sounding, the relationship between the apparent resistivity (ρa) and the layer parameters (e.g. layer thickness, layer true resistivity) can be expressed by an integral equation considering an earth model consisting of homogeneous and isotropic layers. We write the equation following (Koefoed, 1970)

$$\rho_a(s) = s^2 \int_0^{\infty} T(\lambda) J_1(\lambda\lambda s)\lambda d \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where, s is half the current electrode spacing (AB/2) in Schlumberger electrode configuration, J_1 denotes the first-order Bessel function of the first kind and λ denotes the integral variable. Following Koefoed (1970) we write recurrence relationship of the resistivity transform function, $T(\lambda)$ as,

$$T_i(\lambda) = \frac{T_{i+1}(\lambda) + \rho_i \tanh(\lambda h_i)}{[1 + T_{i+1}(\lambda)\tanh(\lambda h_i)/\rho_i)]}, i = n-1,..1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where, n denotes the number of layers, ρ_i and h_i are the true resistivity and thickness of the i^{th} layer respectively.

Inversion scheme

The standard linearized inversion approach to solving the non-linear inverse problems in geophysics has been based on iterative processes. The forward model is developed based on the specific relation between physical models to the observed data. "Inversion" processes update the model parameter in each step to best fit the observed data. However, the inversion of the resistivity sounding data is an ill-posed problem. The reason is being that the contradictory information on model parameter cannot be assessed due to lack of information. Therefore, small changes in the data may lead large changes in the model. Successful optimization depends heavily on the choosing the correct initial model. The problem may be reduced by introducing damping into the system of equations (Roy, 1999). This resulted in a solution of damped least-squares which we can write following (Menke, 1984)

$$\Delta m = (G^T G + \beta^2 I)^{-1} G^T \Delta d \quad \ldots\ldots\ldots\ldots (3)$$

Where Δm is the parameter correction vector; Δd is the data difference vector, G is the Jacobian matrix contains partials derivative of data with respect to the initial model parameters. I is the identity matrix, and the term β is called damping factor which is a scalar quantity actually controls both speed of convergence and solution. This solution is also known as Tikhonov regularization (Levenberg, 1944; Marquardt, 1963; Menke, 1984).

Singular value decomposition (SVD)

Singular value decomposition (SVD) is popular in many areas of geophysical inversion. It is very useful technique for small scale geophysical inverse problem especially, for unstable, rank-deficiency problem. SVD provides numerically stable results in addition to the information related to model parameter resolution and covariance analysis (Meju, 1994).

We factorize an $n \times n$ or $n \times m$ matrix G in the above equation (3) can as follow

$$G = UQL^T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Where for n data and m parameters, $U(n \times m)$ and $L(m \times m)$ are two orthonormal matrix, containing respectively the data space and the parameter space eigenvectors and Q is a $(m \times m)$ diagonal matrix containing at most r non-zero eigenvalues of G, with a condition $r \leq m$. These diagonal entities in matrix $Q\ (\alpha_1, \alpha_2, \ldots\ldots\ldots, \alpha_p)$ are called singular values of G. We write SVD-based damped least squares solution,

containing at most r non-zero eigenvalues of G, with a condition $r \leq m$. These diagonal entities in matrix $Q\ (\alpha_1, \alpha_2, \ldots\ldots\ldots, \alpha_p)$ are called singular values of G. We write SVD-based damped least squares solution,

$$\Delta m = (LQ^2L^T + \beta^2 I)^{-1} LQU^T \Delta d \quad \ldots\ldots (5)$$

We get the form by adding the damping factor to the diagonal elements

$$(LQ^2L^T + \beta^2 I) = \{Ldiag(\alpha_j^2)L^T + \beta^2 I\}$$
$$= Ldiag(\alpha_j^2 + \beta^2)L^T \quad \ldots\ldots (6)$$

We write the inverse of the equation 6 as follows,

$$(Ldiag\{\alpha_j^2 + \beta^2\}L^T)^{-1} = Ldiag\left\{\frac{1}{\alpha_j^2 + \beta^2}\right\}L^T \quad \ldots\ldots (7)$$

Substituting the equation 7 in equation 5 we obtain

$$\Delta m = Ldiag\left\{\frac{1}{\alpha_j^2 + \beta^2}\right\}L^T LQU^T \Delta d \quad \ldots\ldots (8)$$

We obtain the parameter correction vector as:

$$\Delta m = Ldiag\left\{\frac{\alpha_j}{\alpha_j^2 + \beta^2}\right\}U^T \Delta d \quad \ldots\ldots (9)$$

The equation 9 provides damped least-squares solution via the SVD. Usually, in initial iteration, the damping factor set to be a large positive value while making the full use of steepest descent method. Subsequently, at each iteration the damping factor is multiplied by a factor less than unity so that the least-squares methods dominate near the solution (Meju, 1994). Following Maiti et al. (2011) we determine the damping factor as follows:

$$\beta = \alpha_W \Delta c^{\frac{1}{W}} \quad \ldots\ldots (10)$$

Where W is the test number for the damping factor at any iteration, α is the parameter eigenvalue and the term Δc is given by

$$\Delta c_r = \frac{(c_{r-1} - c_r)}{c_{r-1}} \quad \ldots\ldots (11)$$

Where, c_{r-1} is the misfit value obtained at previous iteration and c_r is the misfit found at the current iteration.

In this study, equation 10 and equation 11 were used to set the damping factor in each iteration.

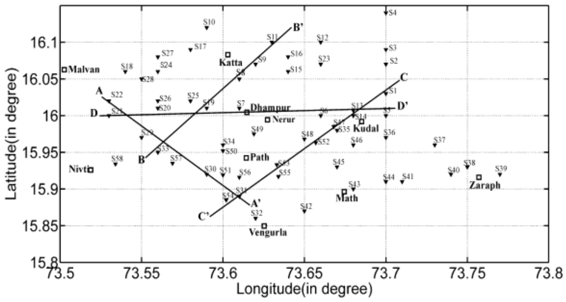

Figure 2: Location map of Schlumberger sounding

Examples

The true resistivity maps obtained by SVD based inversion for depth 1 meter, 10 meter, and 50 meter, are given in Figure 5. According to the map, the study area is displaying almost different features in the vertical direction with increasing depth of penetration. The western and south-western part of the study area clearly shows the low true resistivity zone. The trend of low resistivity is further extended up to the coast of Arabian Sea. Most probably this is the continued sea water intrusion from western and north western side starting from Malvan to Nivti, coastal Maharashtra. Northern and S-E part of the study area clearly suggests high resistive features due partially to lateritic exposure and metamorphic hard rock from surface up to the depth of 10m (Figure 3).

At greater depth of the order of 50m, relatively high resistivity zones are dominated in the N-E and S-E part of the study area. This implies that the tendency of saltwater intrusions is not much in action at deeper depth at eastern part. It is interesting to note that the low resistivities are observed at relatively shallower depth around north-eastern part of the area (Figure 3). The north-eastern part at shallow depth is potential zone for groundwater exploration. It is evident from the true resistivity map that a very high resistivity complex at the surface near Vengurla is a complex metamorphic exposure. The high resistive metamorphic exposure is absent at deeper depth around Vengurla.

Major lineaments are identified in the study area in the direction of N-E and S-W (Figure 3). The lineaments are speculated as the major geological contact where a change of electrical properties must occur. It is observed some criss-cross lineaments. It is worthwhile to note that many lineaments which were identified by previous studies are rejuvenated. The intersection of the lineaments is considered as the possible interesting zone for ground water research of this area (Figure 3).

The 2-D inversion DC electrical resistivity scheme is executed here following the pioneering work by Uchida (1991). It is noteworthy that the one dimensional results provided a good starting model for two dimensional inversions. One can also verify two-dimensional results with the help of one dimensional result and vice versa. In principle 2-D model based interpretation is more appropriate to infer lateral resistivity variation caused by multiple episodes of lava flows and genesis of this area.

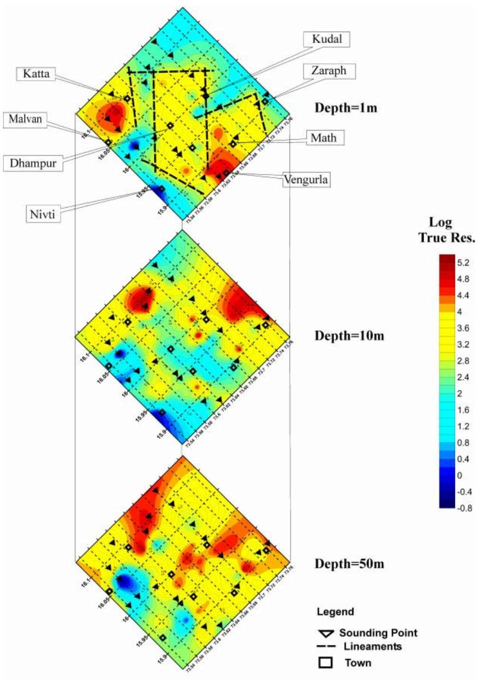

Figure 3: True resistivity contour map at different depth of the study area.

Figure 4 shows one dimensional inversion of Schlumberger vertical electrical sounding (VES) of station S7 and S22 of the study area. The figure indicates that S22 is affected by saline water whereas S7 has no influence of saline water. Figure 5 shows lateral as well as vertical distributions of resistivity along the four profiles (namely, AA', BB',CC' and DD'). Two-dimensional inversion results suggest that surface layer is seldom thick along AA'. Along the profile AA' the low resistive zone is observed just beneath the thin surface layer. This is possibly due to seawater intrusion effect in the station S22 near Malvan, which is near the coast line. This intrusion effect is up to station S30. After S30 station seawater is restricted due to the presence of high resistive rock. True resistivity section is showing an isolated very low resistive area at larger depth surrounded by high resistive feature.

Along profile BB' extending from Nivti to Katta, a high resistive thick granitic layer is observed near the end of the profile. Also there is an isolated low resistivity reservoir surrounded by relatively high resistive formation. Here also the surface layer is thin cover of laterite. There is a potential groundwater reservoir in the station between s19 and s8 which is located in between the North-Western side of the Dhamapur and South-Western side of the Katta (Figure 5). This reservoir is possibly not contaminated by sea water and may fit for the drinking water in the surrounding town because it is a little more away from the western coast. The reservoir is bounded by two major lineaments which are occurred in the direction of N-E to SW. It is possible that the reservoir is recharged via these two major lineaments during the Monsoon. The reservoir is thickened towards S-W side. However it is sharply bounded by the very high resistive metamorphic rock.

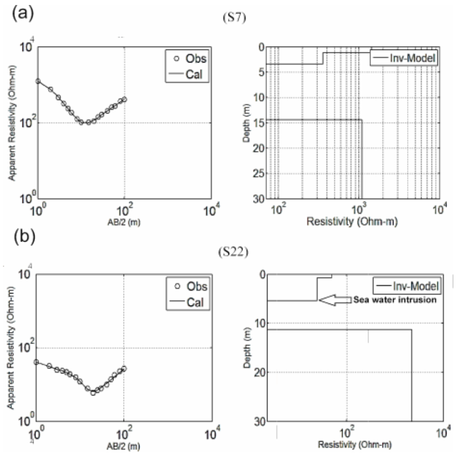

Figure 4: One dimensional inversion of Schlumberger vertical electrical sounding (VES) of station S7 and S22 of the study area. S22 shows the sea water intrusion site.

The 2D-resistivity section CC' covers 7 sounding points from Vengurla to Kudal. Below station S35 isolated reservoir is found which shows low resistivity at depth between 3-5m (Figure 5c)

Consequently below the low resistive feature there is a very high resistive basement layer present at the station s35 (Figure 5). It is also observed that the basement complex is exposed towards Vengurla where it is found a very high resistive metamorphic complex at the surface.

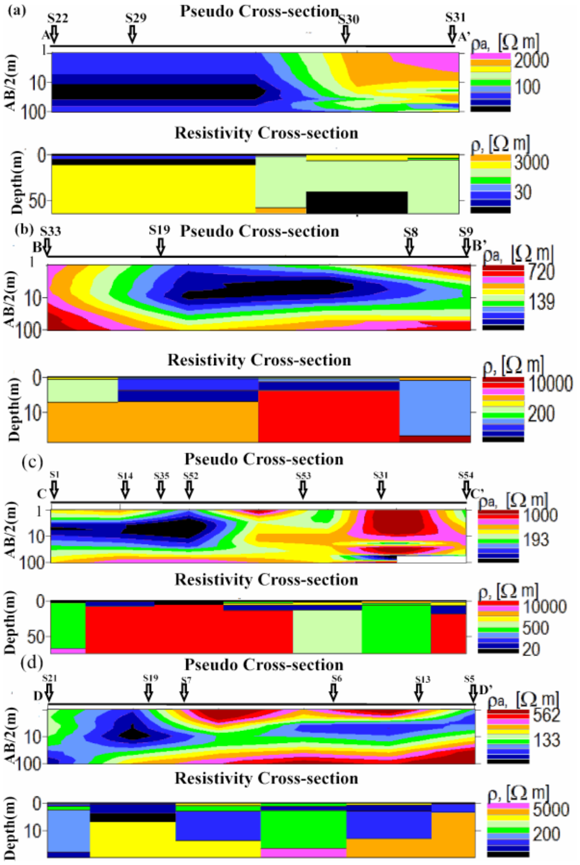

Figure 5: 2D-Resistivity structure along profile (a)AA' (contains sounding point S22-S31); (b)profile BB' (contains sounding point S33-S9) (c) profile CC' (contains sounding point S1-S54);(d) profile DD' (contains sounding point S21-S5)

2-D resistivity cross-section along the profile DD' contains 6- VES sounding point. It is clear from the figure 10 that the conduits is founds along the profile at the depth between 5-10m from Nerrur-Dhamapur up to Kudal town. It is possible that the groundwater flow channel is mainly controlled by the presence of the networks of voids, conduits which connects the major groundwater flow in the Kudal town. It is also possible that along the profile DD' (near S19) the conduits are opened up at deeper depth towards coast (Figure 5).

Thus it is very significant to locate the conduits precisely in the hard rock terrain of Konkan, coastal Maharashtra for possible exploration of groundwater and possible extension of sea water intrusions in the study area (Figure 5).

Conclusions

Singular value decomposition (SVD) based direct current electrical resistivity inversion scheme is developed and applied to the Schlumberger vertical electrical sounding data collected over Malvan-Vengurla region, Konkan, Maharashtra. The inversion method is fast, stable and cost effective. The method based on SVD is found most suitable for small scale rank deficient geo-electrical inversion problem. Delineating the resistivity structure of the coastal area of Maharashtra provides the possible zones infected by the saltwater intrusions from Arabian Sea. The present inversion results suggest that the top layer is mostly comprised of laterites followed by mixture of clay/clayey sand and garnulites/granite/basalts as basement rocks. The conduits are found at the depth of 5-10m from the surface which is responsible for groundwater flow in the hard rock especially in the area of Konkan tract. The networks of voids and conduits are identified to know chief control of the groundwater flow pattern in the hydrological systems of the study area. Narrow buried valleys of coarse sand and gravel areas are not easy to map or sometimes absence in crystalline basement complex where the present method could be an alternative prospect. Prior to application to real data, the method is tested on the synthetic data set and then applied to a large number actual DC resistive profile from coastal zones of Maharashtra for mapping the resistivity structure. Besides providing some new and detail information, the present new results also corroborate well with existing results of borehole geophysics. Thus the method provides a prospective means for delineating resistivity structure of the coastal area of Maharashtra, which in turn, provide the zone of demarcating line of saltwater ingression from Arabia Sea.

References

Ekinci, Y.L., and Demirci., A., 2008. A damped least-squares inversion program for the interpretation of Schlumberger sounding curves. Journal of Applied Sciences 8(22),pp.4070-4078

Ghosh, D., 1971. The application of linear filter theory on the direct interpretation of geoelectrical resistivity sounding measurements; Geophysical Prospecting, 19, pp.192-217

Kirkpatrick, S., Gelatt Jr C D and Vecchi, M. P., 1983. Optimization by simulated annealing, Science, 220,pp.671-680.

Koefoed, O., 1970. A fast method for determining the layer distribution from the raised kernel function in geoelectrical soundings. Geophysical Prospecting, 18,564-570.

Levenberg ,K., 1944. A method for the solution of certain non-linear problems in least-squares, Q. Applied Math, 2,164-168

Marquardt, D.W., 1963. An algorithm for least-squares estimation of non linear parameters, Journal of the Society for Industrial and Applied Mathematics, 11:431-441

Meju, M. A., 1994. Geophysical data analysis: Understanding Inverse Problem Theory and Practice, Society of Exploration Geophysics Course Notes Series, No. 6, 1st Edition, SEG publishers, Tulsa, Oklahoma,ISBN,1-56080-027-5,pp:296

Menke, W., 1984. Geophysical data Analysis: Discrete Inverse Theory, Academic Press, Inc.,New York

Maiti, S., Gupta, G., Erram,V.C., and Tiwari, R.K., 2011 Inversion of Schlumberger resistivity sounding data from the critically dynamic Koyna region using Hybrid Monte Carlo-based neural network approach. Nonlinear Processes in Geophysics, vol.18,:179-192,doi:10.5194/npg-18-179-2011

Rubinstein R.Y., 1981. Simulation and the Monte Carlo method (New York: John Wiley and Sons) 278pp

Roy, I.G., 1999. An efficient non-linear least-squares 1D inversion scheme for resistivity and IP sounding data, Geophysical Prospecting, 47,527-550

Song, S.H, Lee, J.Y and Park, N., 2007, Use of vertical electrical soundings to delineate seawater intrusion in a coastal area of Byunsan, Korea, Environmental Geology, 52, 1207-1219

Zohdy A.R., 1989.A new method for automatic interpretation of Schlumberger and Wenner sounding curves; Geophysics, 54, pp.245-253.