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Multifractal Analysis of Seismicity of Kutch Region (Gujarat)

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Summary

The geographical distribution of past earthquakes is not uniform over the globe. Also, in one region small earthquakes may occur, whereas in another region bigger one may take place. This aspect of earthquake occurrence is covered by the term seismicity. In simple words, seismicity is an expression of the proneness of a region to earthquake occurrence in the historical past including expectations for the future. A region experiencing more frequent and large earthquakes has a higher seismicity compared to one with less frequent and small earthquakes. So in all the spatial and temporal distribution of earthquake in a region is called seismicity of that region. Therefore, the objective of this work is to study the temporal or spatial variation of seismicity by using fractals. The study of temporal variation in D_q *or* D_q *spectra may be used to study the changes in the seismicity structure before and after the occurrence of large earthquakes, which may prove to be of value in forecasting such events. In the present study, we investigate the temporal behavior of generalized dimension* D_q *, b-value as well as* D_q *spectra for earthquakes in the Kutch region of Gujarat for the period 2001-2010 to see if they can serve as a precursory measure.*

Keywords*: Earthquakes, epicenter, generalized dimension, multifractal, magnitude*

Introduction

Earthquakes are the result of fracture processes occurring in rocks. Fractures exhibit a fractal structure over a wide range of fracture scale i.e. from the scales of micro fracture to megafaults (Aki,1981; Brown and Scholz 1985; Turcotte,1986,b; Scholz and Aviles,1986).Fractals are the objects which are having same structure at small scale as well as at large scale. The physical laws governing the fractal structures are scale invariant in nature. Thus fractals are scale invariant which shows dependence on 'Power Law' which is given by -the number of fractures that are larger than a specified size (i.e, Log N= a-bM) and is expressed by the fractal dimension D. Seismicity have power law in time and magnitude also which is expressed by Omori's exponent p and b-value..The fractal structures may be homogenous or have multiscaling. The fractal sets having multiscaling are heterogeneous are called multifractal sets. Such fractals are characterized by generalized dimension Dq. Recent studies have shown that many natural phenomenons such as the spatial distribution of earthquake, fluid turbulence are heterogeneous Multifractals. The heterogeneity and multiscaling of fractal structure of the spatial distribution of the earthquakes in a region is related to the heterogeneity in the distribution of seismicity. A number of studies have been made to investigate the temporal variation of heterogeneity in seismicity using multifractal analysis in various seismic regions (Hirata and Imoto,1991).Recently it has been shown that the spatial and temporal variation in an area may be associated with the process of generation of large size event i.e, a great earthquake (Telesca et al., 2003a,b) seismicity in an area .Therefore the study of temporal variation Dq and Dq spectra may be used to study the changes in the seismicity structure before the occurrence of large earthquake which may be prove to useful in seismicity analysis of an area and in forecasting occurrence of large earthquakes.

The coefficient of the frequency magnitude relationship represents an important parameter for studying the seismicity in certain area. The increase in the b-value in the region is interpreted as the increase in the probability of occurrence of small magnitude events and vice versa for decrease in b-value .The decrease in b-value before a large earthquake is a well observed phenomenon. The

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information of cluster is also observed phenomenon before a large earthquake which means a decrease of fractal dimension of spatial distribution of earthquakes (Ouch et al;1987).The observation suggest that there may be a positive correlation between b-value and fractal dimension (Guo & Ogata 1995; Legrand 2002; Pascuaet al. 2003; Oncel & Wilson 2004). However some observations show negative correlation between the b-value and the fractal dimension (e.g. Hirata 1989;Henderson et al. 1994; Oncel et al. 1996; Wang & Lee 1996). In their experiment, although the fractal dimension decreases with the progress of creep, the decease of b-value was observed only just before the ultimate fracture of rock sample. Therefore there may not be any evidence against positive or negative correlation between b value and the fractal dimension of spatial distribution.

Data Analysis and Methodology

Data analysis: The time window for analysis was chosen to be 2001 to 2010 and IRS earthquake catalogue for the Kutch region for this period was used for multifractal analysis. The earthquakes for the period under investigation are shown in Figure 1.1.The completeness of the catalogue was verified using Guttenberg & Richter relationship and also by Maximum likelihood method and it is found to be complete for magnitude threshold of mb≥1.5.Under the assumption that geological processes are steady over historical time and the observation capabilities are steady over time window under consideration for mb≥1.5, the temporal changes in the frequency of earthquakes may be attributed to the aftershocks of the larger earthquakes and to local temporal variation in seismicity.

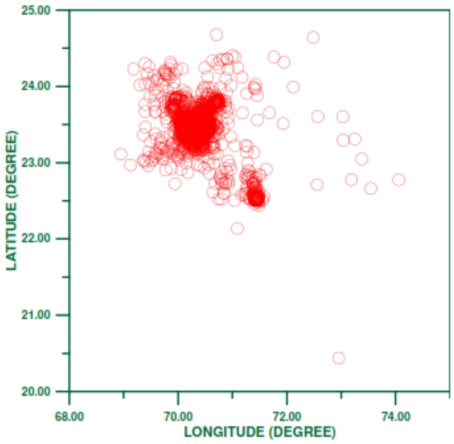


Figure 1.1.Epicentral distribution of Earthquakes occurring in period 2001-10

Completeness of data: In order to obtain unbiased and homogenous data set, catalogue is restricted by setting a lower limit and an analysis time period. This is known as complete of data or we can say that completeness of catalogue.

This is done by plotting no. of events with their magnitude on x-axis as shown in figure 1.2(b).It is found that the data is complete for magnitude M≥1.5.This value of magnitude is threshold value above which we have complete no. of events in the given time window.

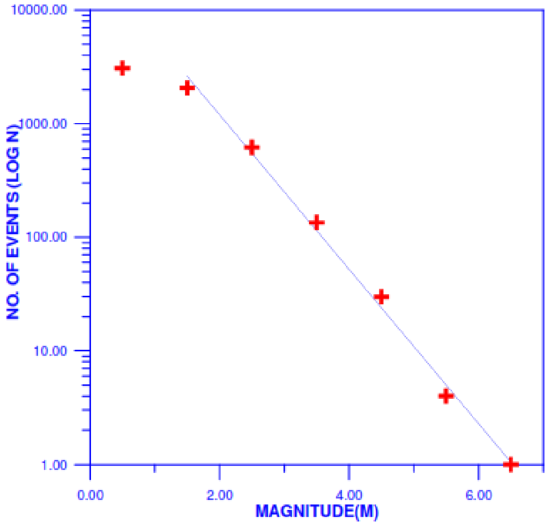


Figure: 1.2(a) Completeness of data plotting no. of events in a particular year and no. of events ≥ in that particular year.(b)completeness of catalogue by frequency-magnitude relationship for M≥1.5

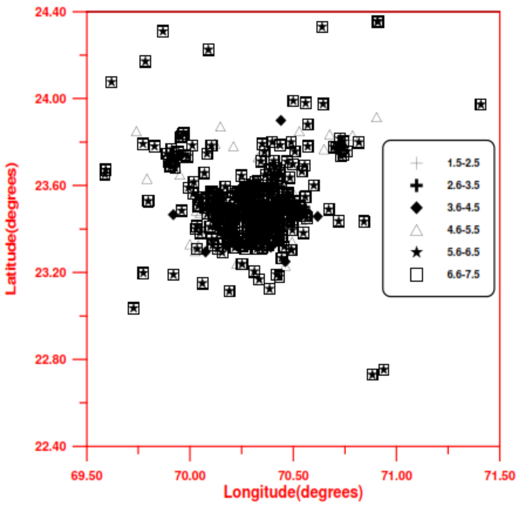


Figure 1.3 Seismicity of selected region for M≥1.5

There are various methods for calculating Dq are fixed Mass method, the fixed radius method Box-counting method (Mandelbrot,1989;Grassberger and Procacia,1983; Hasley et al., 1986; Grassberger et al., 1988;Huang Liji, 1990) These methods work well provided the number of data points is very large. The extended Grassberger and Procaccia method is used in our analysis, which can recover the dimensions from a time series. It is described as follows:

$$Log\,C_q(r) = D_q Log\, r(r \to 0) \cdots\cdots\cdots\cdots (1)$$

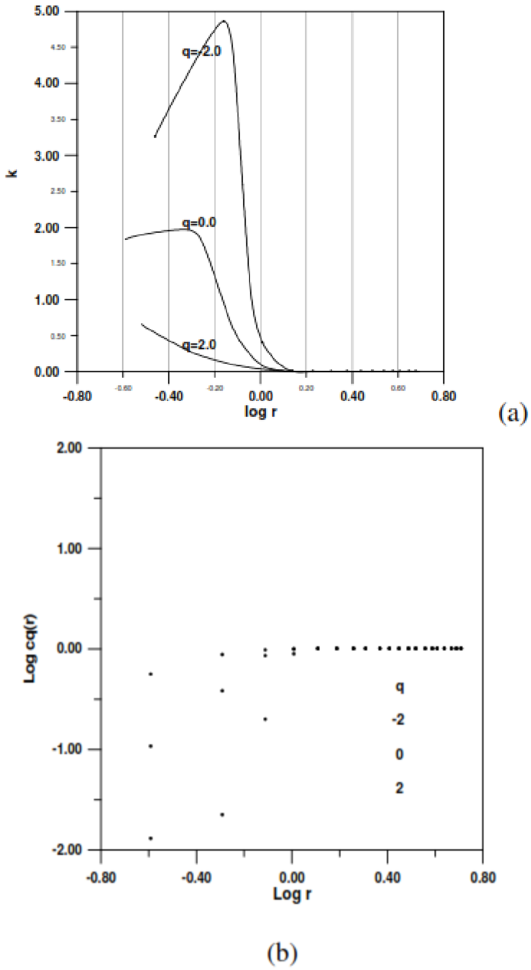
$$C_q\,(r) = \frac{1}{q-1} \lim r \to \left\{ \frac{1}{N}\sum_j^N \left[\frac{1}{N}\sum_{\substack{i=1 \\ i\neq j}}^N H(r - Xi - Xj) \right] \right\}^{q-1} \cdots\cdots\cdots\cdots (2)$$

Where r is the scaling radius, N is the total number of data points within search region in a certain time interval (also called the sample volume); X_i is the epicentral location (given in latitude and longitude) of the $i^{\tau n}$ event X_j is the epicenter (given in latitude and longitude) of the $i^{\tau n}$ event, $C_q\,(r)$ is the $q^{\tau\pi}$ order integral and H (.) is the Heaveside step function.

In the procedure for estimating D_q spectra as a function of time, a time series of earthquakes epicenters has to be formed and divided into sub-series (subsets). Let set $\{X_i\ M_i\}_{i\,=\,1}^{N}$ be a complete set of earthquakes occurring in the time –period 2001-2010, and M_i the magnitude of an earthquake occurring at time τ_i . Thus the earthquakes constitute a time series of n elements. We consider this time series as original data set.

In the present case the original data set is divided into 28 subsets. Each subset consists of 300 events with an overlapping of 100 events. The subsets and their corresponding time are given in Table 1. $C_q\,(r)$ is calculated using equation (2) for the epicentral distribution X_i of the subset. The distance r between two events is calculated by using spherical triangle. For epicentral distribution having a fractal structure, the following power law relationship is obtained in the scaling region.

$$C_q\,(r) \sim r^{D_q} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$



(Figure:1.4 Selection of scaling region for Kutch, N=300.(a) Log r versus Log $C_q\,(r)$ for three values of D(-2,0,2); (b) Log r versus k curve of (a), From(b), scaling region is found to be 0.7.

Table1: The subsets and their corresponding time periods, fractal dimensions along with the number of large events occurring with magnitude M≥4.0

Sr. No	Subset No.	No. of events	Time-period	D-2±S.D.	D2±S.D.	b-value	M≥ 4.0
1	S1	1-300	2000.98-2001.91	1.297±0.280	0.120±0.009	0.24	2
2	S2	101-400	2001.43-2002.97	1.520±0.948	0.088±0.0002	0.29	7
3	S3	201-500	2001.81-2007.09	1.867±0.513	0.137±0.003	0.23	3
4	S4	301-600	2001.91-2007.19	1.959±0.536	0.192±0.006	0.24	3
5	S5	401-700	2003.02-2007.26	1.900±0.285	0.233±0.004	0.26	2
6	S6	501-800	2007.09-2007.33	1.898±0.218	0.209±0.004	0.33	1
7	S7	601-900	2007.19-2007.41	1.926±0.204	0.195±0.004	0.32	2
8	S8	701-1000	2007.26-2007.50	1.823±0.141	0.178±0.004	0.33	2
9	S9	801-1100	2007.33-2007.61	1.949±0.194	0.178±0.003	0.32	2
1	S10	901-1200	2007.41-2007.72	1.978±0.364	0.157±0.003	0.32	1
1	S11	1001-1300	2007.50-2007.80	1.384±0.171	0.148±0.003	0.33	-
1	S12	1101-1400	2007.60-2007.93	1.753±0.244	0.186±0.004	0.36	-
1	S13	1201-1500	2007.72-2008.23	1.745±0.243	0.227±0.006	0.40	-
1	S14	1301-1600	2007.80-2009.02	1.732±0.239	0.313±0.001	0.38	-
1	S15	1401-1700	2007.93-2009.06	1.368±0.164	0.266±0.008	0.43	-
1	S16	1501-1800	2008.24-2009.11	1.514±0.213	0.228±0.006	0.46	-
1	S17	1601-1900	2009.02-2009.15	1.591±0.199	0.151±0.002	0.40	-
1	S18	1701-2000	2009.06-2009.21	1.595±0.208	0.136±0.002	0.39	-
1	S19	1801-2100	2009.11-2009.17	1.845±0.205	0.140±0.001	0.39	-
2	S20	1901-2200	2009.15-2009.35	1.934±0.212	0.147±0.002	0.42	-
2	S21	2001-2300	2009.21-2009.44	1.969±0.191	0.161±0.002	0.40	-
2	S22	2101-2400	2009.27-2009.53	2.028±0.301	0.151±0.002	0.39	-
2	S23	2201-2500	2009.35-2009.64	1.976±0.213	0.134±0.001	0.38	1
2	S24	2301-2600	2009.44-2009.74	2.012±0.282	0.132±0.001	0.39	1
2	S25	2401-2700	2009.53-2009.79	1.996±0.340	0.106±0.001	0.46	-
2	S26	2501-2800	2009.64-2009.84	1.855±0.300	0.113±0.001	0.47	-
2	S27	2601-2900	2009.74-2009.90	1.806±0.298	0.103±0.001	0.49	-
2	S28	2701-3000	2009.79-2009.94	1.792±0.281	0.107±0.001	0.46	-

An appropriate scaling region has to be estimated before the computation of the generalized dimension D_q The scaling region is a linear segment in the graph of Log r versus Log $C_q\,(r)$ in figure 1.4(a).The scaling region may be characterized by the circular boundary defined by scaling radius around epicenter. We use the method Li et al to determine the scaling region in Kutch. Three point curves for q = 2.0.0.0. - 2.0 respectively are shown in figure 1.4(a) which are studied for the selection of the scaling region. There may be two or more linear segments in the each curve. It is known that slopes of adjacent points in the graph will be constant when these points lie on a linear segment. The graph of $Log\,(r)$ versus (is the slope of adjacent points of the graph of $Log\ r$ versus $Log\ C_q(r)$ is shown in figure 1.4(b).The scaling region corresponds to the range in r for which the variation in slope K is minimum. The variation in K for r (0.0^0,0.7^0) is a minimum. There is approximately on linear segment in each curve for different values of q in the graph of Log r versus Log $C_q\ (r)$ is shown in figure 1.4(b).

Result and Discussion

The Dq Vs q is termed as Dq spectra. In this study we have discussed the typical features of Dq spectra for the time period 2001-2010. The Dq spectra may be decreasing or increasing function of q. The shape of Dq spectra is expected to be diagnostic of temporal changes of seismicity pattern before a large earthquake if Dq spectra will decrease with q. The slope of Dq spectra becomes gentle for the extended distribution of earthquakes and it becomes steep for the concentrated distribution of earthquakes. As before a large earthquake clustering of earthquake may occur leading to steep Dq spectra which may serve a precursory anomaly. Thus seismicity distributions have evolutionary nature which may form distinctive pattern. For studying the nature of evolving seismicity pattern in addition to Dq spectra, we have also plotted epicentral locations of events occurring particular events in terms of Latitude & Longitude**.** The consistent decrease in spatial dimension (D_s) may correspond to introduction of clustering of seismicity in the region shows the region preparedness for the occurrence of major events (Hirata et al. 1989). Hirata (1989) reported the negative correlation between b-value and fractal dimension, based upon the observational seismicity data of Tohoku region in Japan. These studies and our study of Kutch region shows similar behavior in spatial temporal variation of fractal dimension as the seismicity in the region evolves from extended distribution of epicenters to clustered distribution and again back to distributed seismicity. This evident from subsets S15 to S28. In general, earthquake of magnitude 4.1 can be related to evolving of self organized critical behavior of the seismicity which results in occurrence of earthquakes of magnitude M≥4.0.

Observation of seismicity suggests a relationship between the distribution of earthquake magnitude and the distribution of earthquake epicenters. The fractal dimension D_2, D_{-2} for all subsets (S1-S28) is shown in Table 1.The temporal variation of generalized fractal dimension D_q (q=2, 0,-2) for all 28 subsets for the time period 2001 to 2010.The Dq values for all 28 subsets are plotted at time where the last earthquake entered in the subset (see figure 2.2).The first subset of 300 earthquakes starts 2001.98 years and ends at 2001.91 years. Thus the Dq values evaluated for all subsets are plotted on time axis for all subsets. In general, D_{-2} varies from 1.3 to 2.0; D_2 varies from 0.08 to 0.2 and D_0 Varies from 0.5 to 0.9. *b-value:* The temporal variation of b-value is also studied from the time period 2001-2010.The variation in b-value ranging from minimum value 0.10 to maximum value 0.43 is shown in Table 1.The relationship between D2 and b-value is also analyzed for all subsets as shown in figure 2.3(a). However it is more scatter more all subsets but it is showing positive relationship for 23 subsets (see figure 2.3(b)). The consistent and significant decrease in Dq & b -value has been observed prior to occurrence of large earthquake. We note that changes in b-value as well as D2 from subset S15 (2007.93-2009.06) to S22 (2009.21-2009.44) and significant increase in D2 and decrease in D2 (see figure 2.1). The earthquake of magnitude M≥ 4.0 occurred in subset S23.

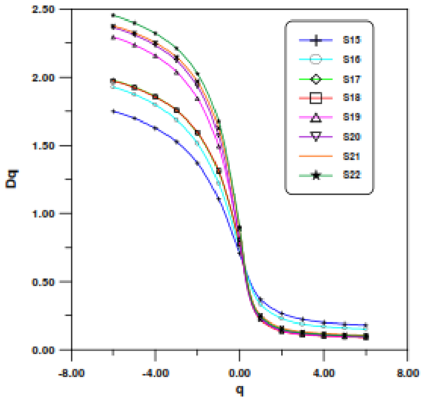


Figure 2.1:Dq spectra of subsets 15-22

In this study when we compare the Dq spectra of subset S15 with S22, we found that Dq spectra of subset S22 is steeper as compared to subset S15 as shown in figure 2.2 (a) which results in clustering of earthquake before a large size earthquake. Clustering of earthquakes is also confirmed by plotting the epicentral locations of these two subsets as shown in figure 2.4(b), (c). Moreover we have compare the Dq spectra of subsets between S15 to S22 which results in steep slope of Dq spectra from S15 to S22 i.e., decrease in value of Dq with q as shown in figure 2.4(a)

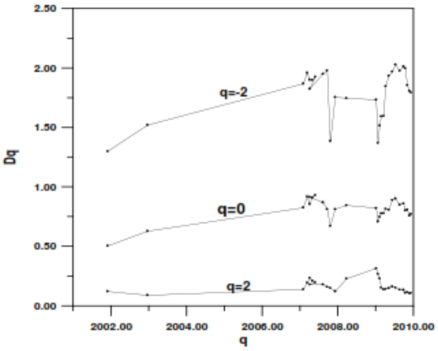


Figure2.2: Plot of D2 Vs q of 1-28 subsets

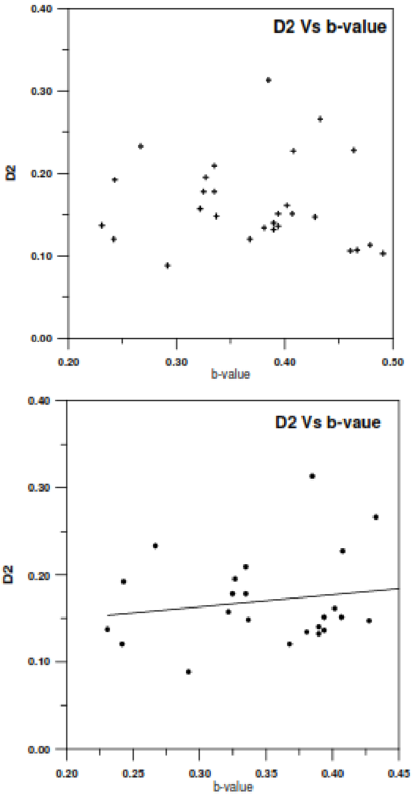


Figure 2.3(a) Plot of D2 Vs b-value of all subsets (b) for 23 subsets

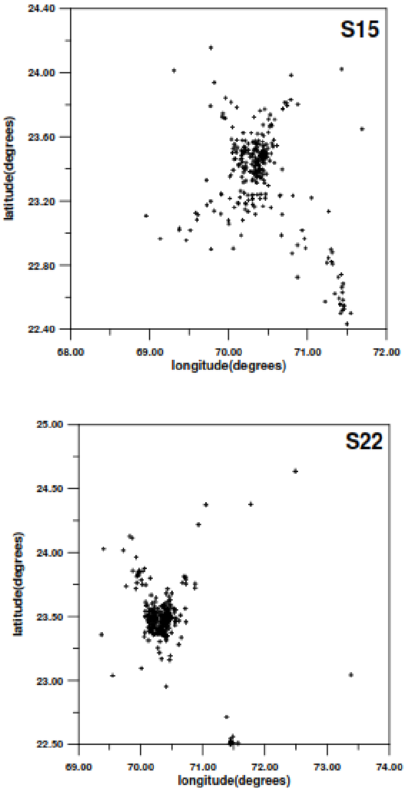


Figure 2.4(a): Dq spectra of subset 15 and subset 22 ;(b), (c): Epicentral location of events occurring subsets 15 &22

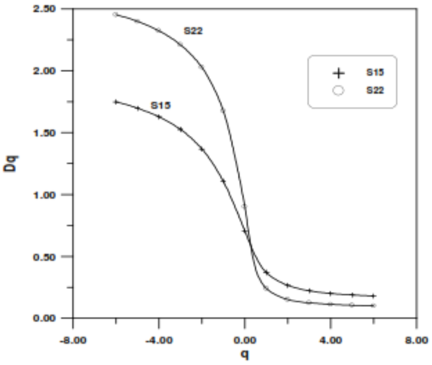


Table2: Showing subsets with minimum and maximum Value of magnitude occurring in each subset with b-value

Subset	Min. Magnitude	Max. Magnitude	b-value
S1	1.1	6.9	0.24
S2	1.1	4.7	0.29
S3	0.8	5.7	0.23
S4	0.8	5.7	0.24
S5	0.6	5.7	0.26
S6	0.6	4.4	0.33
S7	0.6	4.1	0.32
S8	0.7	4.2	0.33
S9	0.7	4.2	0.32
S10	0.7	4.2	0.32
S11	0.8	4.1	0.33

S12	0.8	3.8	0.36
S13	0.8	3.8	0.40
S14	0.5	3.8	0.38
S15	0.5	3.8	0.43
S16	0.5	3.3	0.46
S17	0.4	3.6	0.40
S18	0.4	3.6	0.39
S19	0.4	3.7	0.39
S20	0.5	3.7	0.42
S21	0.5	3.7	0.40
S22	0.5	3.3	0.39
S23	0.5	4.1	0.38
S24	0.5	4.1	0.39
S25	0.6	4.1	0.46
S26	0.6	3.6	0.47
S27	0.6	3.5	0.49
S28	0.6	3.5	0.46

Conclusions

The seismicity data of studied region show Multifractal behavior. Multifractal analysis of seismicity data holds promise for serving as precursory parameter for earthquakes data sets having lower magnitude threshold and larger time window. The spatio-temporal variation of Dq and Dq-q spectra provides some lessons to learn from nature. This study supports the use of spatio-temporal variation of Dq at small q (i.e,-2, 0, and 2) to detect the seismicity change in regional area. The steep slope in Dq-q spectra is detected before the occurrence of large earthquake. However the steep slope may not necessarily be due to increase in Dq for negative values of q and decrease in Dq for positive values of q when compared from one subset to consecutive subset in case seismicity is converging to very localize distribution i.e., from randomized extended distribution to self –organized clustered distribution of epicenters. In general, earthquakes of magnitude 4.1 can be related to evolving self organized critical behavior of the seismicity which is resulted by the occurrence of earthquakes $M\geq4.0$.Athough the catalogue is complete for magnitude $M\geq1.5$ but the region is complete since 2007 onwards.

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