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# Effective medium modeling of quasi-elastic medium

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***Summary***

*The objective of this study is to account for squirt flow in the computation of effective elastic parameters of a heterogeneous rock. We review the existing rock physics models that consider fluid interaction during wave propagation and then describe a new effecting medium modeling keeping in mind the limitations of the previous theories. The new model is based on the Eshelby's inclusion model and the differential effective medium (DEM) theory. We have shown a numerical example to model a complex heterogeneous medium using the modified DEM theory. The new model predicts well the elastic properties of* $CO_2$*-squestered carbonate and sandstone rocks that vary in textures and are prone to chemical reactions with the* $CO_2$*-rich water.*

***Keywords****:* $CO_2$ *sequestration, modified differential effective medium theory, squirt flow, quasi elastic properties*

## Introduction

The microstructure of a rock is heterogeneous, because of different shapes, sizes and orientations of cavities, which causes the wave-induced pressure gradient (Figure 1).

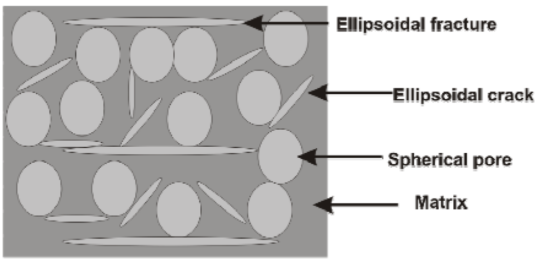


Figure 1: Schematic diagram of microstructure of a rock

There are two types of fluid interactions during wave propagation: (1) Global Flow, caused by pressure gradients at the scale of the wavelength and in the direction of wave propagation and (2) Squirt Flow, caused by pressure gradients at the scale of the microstructure and in the directions that are generally different from that of wave propagation. The existing effective medium theories can be grouped into two categories: phenomenological and inclusion-based models (Jakobsen and Chapman, 2009). In the phenomenological type (Biot 1956, 1962; Dvorkin and Nur, 1993; Mavko and Jizba, 1991; Mukerji and Mavko 1994; Dvorkin et al., 1995; Sil et al., 2010), the model parameters are empirical and are not related directly to the details of the microstructure, whereas inclusion based models (Mavko and Nur, 1975; O'Connell and Budianski, 1977; Hudson et al., 1996; Jakobsen et al., 2003b; Jakobsen and Hudson, 2003; Jakobsen, 2004; Chapman et al., 2002; Chapman, 2003; Jakobsen and Chapman, 2009) are typically based on Eshelby's inclusion model and consider microstructure of a rock. The phenomenological approach that Biot (1956) established is attractive when dealing only with the global flow. The inclusion-based approach is more appealing when squirt flow, sensitive to the details of the microstructure, is included. Recently, Jakobsen et al. (2003a) extended the T-matrix approach to squirt flow mechanism (Jakobsen 2004, Jakobsen and Chapman, 2009). Chapman et al. (2002) and Chapman (2003) have used Eshelby's (1957) interaction energy approach to account for squirt flow in fractured porous media, where background is taken as isotropic containing a single mineral. In many geological situations, the background medium may be anisotropic and heterogeneous containing more than one mineral. Also, how the partial alignment of cracks, which is more realistic than complete alignment, can be accounted for, is not clear from this modeling. Though an inconsistency or error related to fluid mass conservation present in the squirt flow model in the original work of T-matrix approach (Jakobsen 2003b) is corrected in the unified theory of global flow and squirt flow (Jakobsen and Chapman, 2009), complete investigation on this topic is yet to come (Jakobsen and Chapman, 2009).

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Our aim is to find dynamic (frequency-dependent) elastic moduli of a medium for all possible ranges of physical parameters taking into account the limitations of the existing models.

## Theory

In this work we have introduced a new model based on Eshelby's inclusion model and differential effective medium theory (DEM).

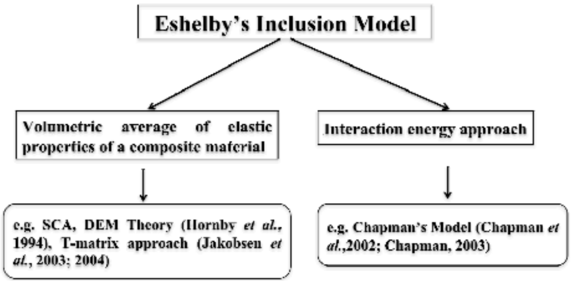


Figure 2: Different approaches of Eshelby's inclusion model

Eshelby (1957) established that the strain due to a homogeneous applied stress at infinity inside an inclusion within an infinite, homogeneous matrix is also homogeneous and calculated the response of a single ellipsoidal inclusion. There are two approaches to using Eshelby's inclusion model (Figure 2) in building effective medium models of a fractured, porous rock.

DEM considers a preferred matrix or host material and inclusions. In case of two-phase composites, volume of the inclusion phase (phase 2) is increased infinitesimally replacing the matrix material (phase 1) until the desired concentration is reached. At each step of increment, the new effective medium becomes the host material (Nishizawa, 1982; Berryman, 1992, Hornby et al., 1994). In this way, the number of phases can be increased.

The compliance of effective material (S*) that we wish toestimate can be given as (Hornby et al., 1994)

$$S^* = S^0 - \sum v_i (S^0 C^i - I) K^i \ , \qquad (1)$$

where, $S^0$ is the compliance of the matrix material, $C^i$ is stiffness of the ith inclusion and $v_i$ is the volume concentration of the ith inclusion.

The inclusion strain is given by (Hornby et al., 1994)

$$e^{inc} = K^i \sigma \ . \qquad (2)$$

Therefore, equation (1) can be rewritten as

$$S^* = S^0 - \sum v_i (S^0 C^i - I) e^{inc} (\bar{\sigma}) \ . \qquad (3)$$

### Inclusion at constant hydrostatic pressure

If the effective stiffness of a medium is $C$, and average stress and strain are respectively $\sigma$ and $e$, then the stress in inclusion is given by

$$\sigma^{inc} = \sigma + C(e^c - \varepsilon^*) = C^i (e^c + e) \ , \qquad (4a)$$

where, $\varepsilon^*$ is the eigen strain and

$$e^c = PC\varepsilon^* \qquad (4b)$$

$$P = 1/8\pi (G_{ikjl} + G_{jkil}) \ . \qquad (4c)$$

The strain in inclusion is given by

$$e^{inc} = e^c + e \quad . \qquad (5)$$

At a constant hydrostatic pressure, the stress in inclusion is equal to the pressure tensor $p_f \delta$ (Zatsepin and Crampin, 1997; Chapman, 2002). Therefore, the inclusion stress is given by

$$\sigma + C(e^c - \varepsilon^*) = p_f \delta \ , \qquad (6)$$

where, $\delta$ is the Dirac delta function.

Combining equations (4), (5) and (6), we obtain an expression for the strain in inclusion at constant hydrostatic pressure as

$$e^{inc} = (I - PC)C^{-1}(\sigma - CPp_f \delta) \ . \qquad (7)$$

Substituting equation (7) in (3) and reformulating as a differential equation (Hornby et al., 1994) in stiffness, the final form of the modified DEM theory is given as

$$\frac{dC}{dv} = \frac{1}{1 - v_i}(C_i - C)(I - PC)^{-1} C^{-1} (\bar{\sigma} - CPp_f)\bar{\sigma} C \quad . \qquad (8)$$

In our modeling, we have assumed the distribution of cavities to be the same as that of Chapman (2003) and thus the pressure in each inclusion is calculated in the similar way as Chapman (2003). Average elastic stiffness for the random/partial/complete alignment of cracks/fractures is computed by combining Voigt-averaging and the method of smoothing (Jakobsen et al., 2000; Ghosh et al., 2010).

## Numerical examples

The modified DEM theory can be used to calculate dynamic elastic parameters for the components (pores/cracks/fractures) having small to large aspect ratios (i.e. any geometry), and crack/fracture density even greater than 0.1 (specially in hydrocarbon reservoirs). In this work, fracture is defined as cavities much larger than the cracks.

| Cavities | Volume fraction/ Density | Aspect ratio | Radius/ length (m) |
|---|---|---|---|
| Spherical pores | 0.10 | 1 | 0.0001 |
| Thin cracks | 0.095 | 0.002 | 0.0001 |
| Long fractures | 0.16 | 0.01 | 0.1 |

Table 1: Parameters used for different cavities in modeling.

***Numerical example.*** We have assumed an isotropic background medium with spherical pores and randomly oriented cracks. The resulting medium is isotropic. Horizontally aligned fractures are added in the cracked-porous medium. Now, the effective medium is vertically transversely isotropic (VTI). The Lame' parameters are taken as $\lambda$ = 17.5 GPa and $\mu$ = 17.5 GPa. The characteristic time for squirt flow is taken as $\tau_c = 2x10^{-6}$ s. Bulk modulus of water is taken as 2.2 GPa. For any visco-elastic fluid, shear modulus of the fluid is $i\omega\eta$ instead of zero (Walsh, 1969), where, i denotes the imaginary constant, $\omega$ is angular frequency and $\eta$ is the viscosity of water taken as 1 cP (Chapman *et al*., 2002). The parameters of cavities used for modeling are given in Table 1.

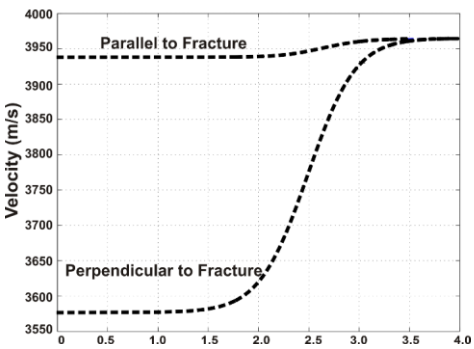


Figure 3: P-wave dispersion curve

Figure 3 shows the P-wave dispersion curves for this VTI medium. P-wave parallel to the fractures is almost independent of frequency but P-wave propagating normal to the fracture is dispersive and dispersion starts below 100 Hz, which is significant.

Comparison with real data. We apply the pressure dependent DEM theory to two different types of data sets:

### A. Ultra-frequency laboratory measurement

$CO_2$-water mixture injected into pure calcite (carbonate) rock where microstructure, porosity and permeability change due to chemical dissolution and/or precipitation of $CaCO_3$ (Vialle and Vanorio, 2011). The porosity changes from 26.49-31.46% and permeability (k) changes from 75.3-448.9 mD. Initially the characteristic time for squirt flow is taken as $\tau_c=2x10^{-7}$ s and then calculated as $\tau_c \propto 1/k$.

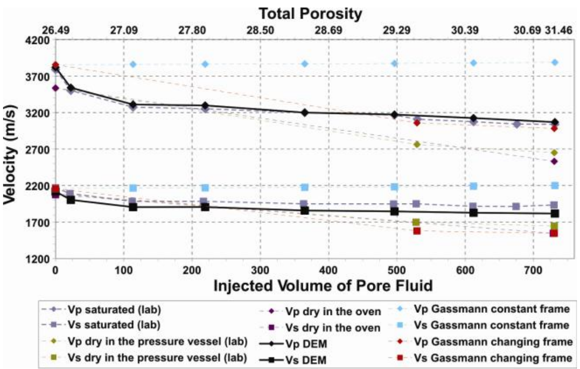


Figure 4: $V_p$ and $V_s$ are predicted by the modified DEM theory and compared with the laboratory measurements (Vialle and Vanorio, 2011) in the same laboratory conditions.

### B. Well log measurement

Sandstone reservoir is a very good sink for $CO_2$, where $CO_2$ may remain in super critical state or dissolved in water. Figure 5 shows the pre-injection sonic velocity of a sandstone reservoir and the predicted velocity from the modified DEM theory using the pre-injection pressure, porosity, permeability log etc. This sandstone reservoir is mainly composed of quartz. Clay minerals are present as matrix material and chlorite as cement material. Figures 6 and 7 show the predicted post-injection velocities for uniform and patchy distribution of $CO_2$ gas respectively. Some amount of $CO_2$ gas is supposed to be dissolved in water, which is chemically reactive. The laboratory experiment on $CO_2$-sequestered sandstone shows cementing material chlorite is dissolved in the $CO_2$-rich water (joy et al., 2011). But porosity and permeability do not change significantly due to overburden pressure. Therefore, while predicting post-injection velocities, we have calculated elastic properties using the post-injection pressure, temperature and fluid density log, and assumed that chlorite cement is dissolved.

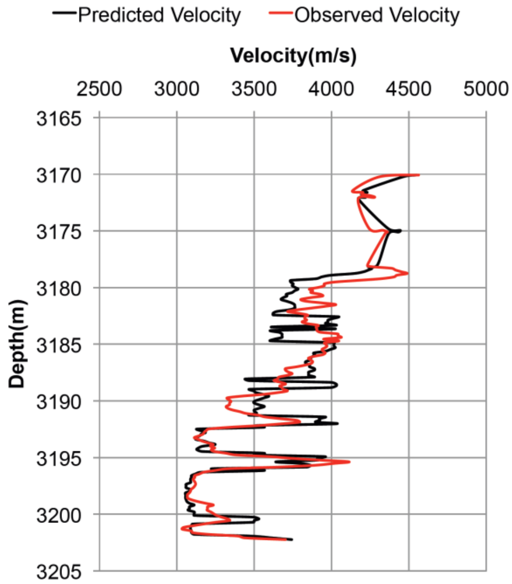


Figure 5: Pre-injection sonic and predicted compressional velocities

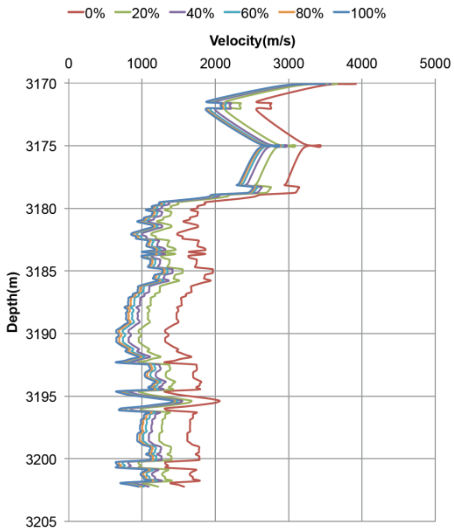


Figure 6. Predicted post-injection velocities at different $CO_2$ saturation for uniform distribution.

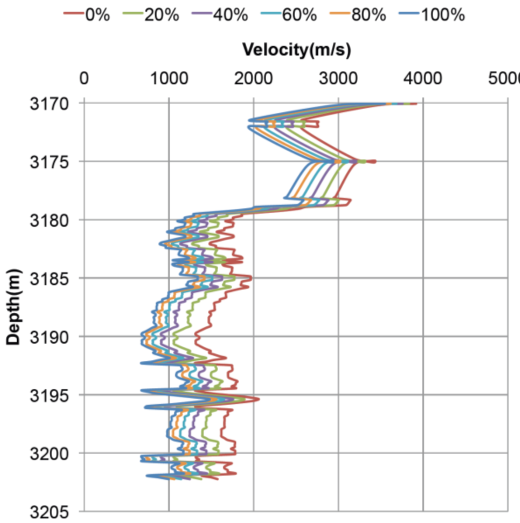


Figure 7. Predicted post-injection velocities at different $CO_2$ saturation for patchy distribution.

## Conclusion

In this paper, we describe the development of a new effective medium-modeling algorithm by extending the existing DEM theory, which is able to model a wide range of geometries of the inclusion (thin vein to spherical nodule). In DEM modeling, the components can be increased to any concentration by increasing infinitesimally at each step. The order of including any component does not necessarily correspond to actual geological evolution. This theory can handle both isotropic or anisotropic background medium, which may contain isotropic or anisotropic inclusions. The pressure dependent modified DEM is capable of accounting for the squirt flow mechanism that makes the theory most general so far.

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