



P-448

Estimation of Chargeability and Permeability using Spectral Induced Polarization Data

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Summary

Induced polarization (IP) method has been successfully implemented in investigation of groundwater, geothermal and metallic ores. From published (Nordsiek and Weller, 2008) spectral induced polarization (SIP) data, the amplitude and phase spectra have been derived. Frequency-dependent impedance data sets have been calculated from these SIP data sets, which were derived for chargeability by Debye Decomposition using non-negative least squares algorithm. Later different parameters (total chargeability, etc.) were calculated from the chargeability and compared with the published one, which shows a good match with published ones. Permeability is also calculated. Synthetics example has been done to check the quality of non-negative least square algorithm.

Keyword: Spectral Induced Polarization, Debye decomposition, Non-negative least squares algorithm, Chargeability and Permeability.

Introduction

Induced polarization (IP) is a geophysical technique, used for searching the disseminated metallic ores. The method is similar to electrical resistivity tomography. The induced polarization method is an active one because voltages, which can be as high as several thousand volts in time domain surveys, are applied to the ground in order to generate measurable over-voltage (Reynolds, 1997). In the time domain, IP effects can be observed by slow voltage decay after switching off the driving current signal and in the frequency domain polarization phenomena cause a phase shift between injected current and measured voltage signal. The frequency-dependent behavior of resistivity amplitude and phase shift is recorded.

IP decay curve can be transformed into relaxation time spectra, which can be used as an in-situ method to estimate pore structure, permeability, formation water resistivity and salinity of reservoir with deep investigation depth and high signal-to-noise ratio (Tong et al., 2004; Tong et al., 2006).

By adopting Debye decomposition for SIP spectra of synthetic samples we derived sequence of relaxation times

and chargeabilities that yield the integrating parameters like total chargeability and mean relaxation time (Nordsiek and Weller, 2008). These parameters are used in a general power-law equation for permeability estimation. There is a wide variation of parameters derived from IP spectra. These parameters strongly depend on the fitting model e.g., Cole-Cole model (Cole and Cole, 1941) and Cole-Davidson model (Davidson and Cole, 1951). Because the model should be chosen in accordance with the special shape of the spectra, the resulting relations are restricted in their applicability. Impart unsuccessful attempt (Nordsiek and Weller, 2008) were made to derive time-domain data from frequency-domain SIP data.

Inversion Methodology and IP Parameters

Different approaches have been proposed for computation of the relaxation-time distribution from time-domain or frequency-domain IP data (e.g., Morgan and Lesmes, 1994; Tong et al., 2006; Tarasov and Titov, 2007; Nordsiek and Weller, 2008). The spectral IP and time-domain IP data inversion is an important step in IP data analysis. The generalized form of the Cole-Cole model (Pelton et al., 1983) describes the frequency dependence of the



Estimation of Chargeability and Permeability using Spectral Induced Polarization Data



impedance $Z(\omega)$ as a function of five independent parameters,

$$Z(\omega) = R_0 \left(1 - m \left(1 - \frac{1}{(1+i\omega\tau)^c} \right)^a \right) \quad (1)$$

Where, R_0 being the resistance for direct current (DC), m is the chargeability, τ is the time constant, and c and a are the exponent. The angular frequency ω is described by $\omega = 2\pi f$, where f is related to the measured frequency.

If a is 1, the resulting model is referred to as Cole-Cole and if c is 1, the resulting model is referred to as Cole-Davidson. In the case that exponents a and c become 1, the model simplifies to Debye. For the Debye model, τ denotes the relaxation time of the exponential decay curve that is used to describe the discharge of a capacitor in an electrical circuit.

In **frequency-domain**, frequency-dependent impedance $Z(\omega)$ that describes the IP spectrum can be presented by the equation.

Debye spectrum in frequency-domain:

$$Z(\omega) = R_0 \left[1 - m \cdot \left\{ 1 - \frac{1}{1+i\omega\tau} \right\} \right] \quad (2)$$

Superposition of n different Debye models leads to:

$$Z(\omega) = R_0 \left[1 - \sum_{k=1}^n m_k \cdot \left\{ 1 - \frac{1}{1+i\omega\tau_k} \right\} \right], \quad (3)$$

Where, n is the number of individual Debye responses. Each Debye response is given by a specific chargeability m_k and relaxation time τ_k .

The n pairs of relaxation time τ_k and chargeability m_k , and the value of the DC resistance R_0 , are the resulting parameters of this model. The DC resistance R_0 is approximated by extrapolating the impedance amplitude to lower frequencies. In an additional step after the determination of all other parameters, the value of R_0 is adjusted using only the amplitude spectrum. To express the problem as a system of linear equations, a simple transformation must be done. The impedance is normalized in the following way:

$$Z_{norm}(\omega) = \frac{R_0 - Z(\omega)}{R_0} \quad (4)$$

Then from equation 3

$$Z_{norm}(\omega) = \sum_{k=1}^n m_k \cdot \frac{(\omega\tau_k)^2}{1+(\omega\tau_k)^2} + i \sum_{k=1}^n m_k \cdot \frac{\omega\tau_k}{1+(\omega\tau_k)^2} \quad (5)$$

On separating the normalized impedance $Z_{norm}(\omega)$ into real and imaginary part,

Real part:

$$Z'_{norm}(\omega) = \sum_{k=1}^n m_k \cdot \frac{(\omega\tau_k)^2}{1+(\omega\tau_k)^2} \quad (6)$$

Imaginary part:

$$Z''_{norm}(\omega) = \sum_{k=1}^n m_k \cdot \frac{\omega\tau_k}{1+(\omega\tau_k)^2} \quad (7)$$

Assuming that at p discrete angular frequencies, the normalized impedance values are determined. A combination of equations 6 and 7 provide the following system of linear equations, consisting of $2p$ equations and n parameters of m_k to be determined. The system of linear equations proposed by Nordsiek and Weller (2008) in matrix form is given by

$$\begin{pmatrix} \frac{(\omega_1\tau_1)^2}{1+(\omega_1\tau_1)^2} & \dots & \frac{(\omega_1\tau_n)^2}{1+(\omega_1\tau_n)^2} \\ \vdots & \ddots & \vdots \\ \frac{(\omega_p\tau_1)^2}{1+(\omega_p\tau_1)^2} & \dots & \frac{(\omega_p\tau_n)^2}{1+(\omega_p\tau_n)^2} \\ \frac{\omega_1\tau_1}{1+(\omega_1\tau_1)^2} & \dots & \frac{\omega_1\tau_n}{1+(\omega_1\tau_n)^2} \\ \vdots & \ddots & \vdots \\ \frac{\omega_p\tau_1}{1+(\omega_p\tau_1)^2} & \dots & \frac{\omega_p\tau_n}{1+(\omega_p\tau_n)^2} \end{pmatrix} \cdot \begin{pmatrix} m_1 \\ \vdots \\ m_n \end{pmatrix} = \begin{pmatrix} Z'_{norm}(\omega_1) \\ \vdots \\ Z'_{norm}(\omega_p) \\ Z''_{norm}(\omega_1) \\ \vdots \\ Z''_{norm}(\omega_p) \end{pmatrix} \quad (8)$$

The n values of relaxation time τ_1 within the matrix are predetermined.

In this study, the n values of relaxation time τ^k within the matrix are predetermined. We consider the results of numerical tests, $n = 64$ equally distributed values at a logarithmic scale from 10^{-4} through 10^3 s.

$n = 64$ (0.0001, 0.00013, 0.00017,, 0599.5, 774.3, 1000)

The “Isqnonneg (**Non-negative least-squares**)” algorithm is used for solving this problem and is implemented in the MATLAB software. Nordsiek and Weller (2008) have proposed integrated parameters. They are as follows:



Estimation of Chargeability and Permeability using Spectral Induced Polarization Data



The total chargeability (m_t): it is given by sum up of all chargeability values m_k

$$m_t = \sum_{k=1}^n m_k, \quad (9)$$

The **mean relaxation-time** $\bar{\tau}$: it equals the logarithmic average values of the relaxation times weighted by their respective chargeability values. It is given by the formula

$$\bar{\tau} = \exp\left(\frac{\sum_{k=1}^n m_k \cdot \ln \tau_k}{m_t}\right), \quad (10)$$

Computation of average relaxation time

Two average time constants i.e. geometric average time constant T_g and arithmetic average time constant T_a , are used to estimate permeability. These are defined as follows, respectively:

$$T_g = \left(\prod_{i=1}^N T_i^{f_i}\right)^{1/\sum_{i=1}^N f_i}, \quad (11)$$

$$T_a = \frac{\sum_{i=1}^N f_i \times T_i}{\sum_{i=1}^N f_i}, \quad (12)$$

Where, T_i are relaxation time constants and f_i are corresponding amplitudes.

Estimation of permeability

According to the proposed model of Tong et al. (2004), relationship between porosity and permeability is given by

$$K_g = c \times (T_g^m \times \Phi^4)^b \quad (13)$$

$$K_a = c \times (T_a \times \Phi^4)^b \quad (14)$$

Where, b and c are constants and they are different for equations (13) and (14).

Since there is no information about the permeability of the samples (Nordsiek and Weller, 2008), it is impossible to correlate the permeability values. Also we do not have any idea of the value of constant b and c , we use the average values of b and c provided in tong et al. (2004) for the estimation of the permeability, which are given as $b = 1.57$ and $c = \exp(-9.6)$.

IP Data Analysis

Our approach to analysis of SIP data is based on the published SIP data (Nordsiek and Weller, 2008). Nordsiek and Weller (2008) have prepared 13 synthetic samples,

over which SIP measurements have been made by SIP-Fuchs equipment (by Radic Research, Berlin). The slag material used in their study was taken from two old smelting places in the Harz Mountains in central Germany, which have been partly excavated by archaeologists (Weller, 2003). The IP data analysis in time-domain and frequency-domain has been done by amplitude and phase spectra (Nordsiek and Weller, 2008). Nordsiek and Weller (2008) have used total 13 samples for SIP data analysis but only the spectra details of sample number 6 and 13 are given in their paper. Figure 1 and 2 shows the amplitude and phase spectra for samples 6 and 13 respectively. So, for present analysis, we have used these spectra to calculate the chargeability. These samples are prepared by slag-sand mixture of different grain size.

Sample properties

Composition and Origin of the investigated samples are shown in table 1.

Table 1: Composition and Origin of the investigated samples

Sample no.	Grain size (mm)	Mass of slag (%)	Porosity (%)	Water saturation (%)	Origin of the slag material
1	<1	10	48	86	Pandebach Valley
2	<1	20	48	87	
3	<1	30	48	88	
4	1-2	30	48	75	
5	2-4	30	43	89	
6	4-6.3	30	43	86	
7	<0.1	30	40	79	Hunenberg
8	0.1-0.5	30	45	85	
9	0.5-1	30	41	97	
10	1-2	30	43	87	
11	2-4	30	39	99	
12	4-6.3	30	40	90	
13	~10	30	36	95	

These samples were prepared manually with slight differences in porosity and water saturation. The conductivity of tap water is approximately 10 mS/m. The slag was crushed and sieved into the grain size ranges for all samples except sample number 13. Amplitude and phase spectra are shown in Figure 1 and 2 for sample



Estimation of Chargeability and Permeability using Spectral Induced Polarization Data



number 6 and 13 respectively. Sample characteristics are included in Table 1 (Nordsiek and Weller, 2008). The amplitude and phase values for sample number 6 and 13 are taken with the help of DigitizIt_win 1.5, software.

Results and Discussion

A further result of the Debye decomposition is the relaxation time distribution, which relates the chargeability values to the time scale between 10^{-4} and 10^3 sec. (Nordsiek and Weller, 2008). For smaller grain size, most of the chargeability originates from lower relaxation-times; and for bigger grain size, most of the chargeability originates from higher relaxation-times (Reynolds, 1997). Chargeability is calculated using non-negative least squares algorithm.

Spectral IP results using non-negative least squares algorithm

Figure 3 and 4 shows the plot of chargeability values versus relaxation time for samples 6 and 13 respectively using non-negative least squares algorithm.

Generally, it is observed that if polarizable mineralization is coarse-grained, the relaxation time is large and the critical frequency is small (conversely for fine-grained mineralization) (Reynolds, 1997). For sample number 6, phase spectra in Figure 1 is broadend with increasing grain size and chargeability plot versus relaxation time in Figure 3 shows that chargeability decreases with increasing grain size at higher relaxation time. But for sample number 13, it is not true. The main reason is, slag grains for sample number 13 were not crushed and slag grains were selected manually according to predetermined size from a large volume of uncrushed slag material. So, the surface of the uncrushed slag grain (sample 13) is smoother than that of the crushed grain (sample 6). The second cause is fractal geometry of the rough grain surfaces of the crushed slag material (sample 6) in comparison with the smooth natural surface of the grain for sample 13 (Nordsiek and Weller, 2008).

Figures 5 and 6 show normalized chargeability plotted versus relaxation time as a cumulative curves for sample number 6 and 13 respectively. Figure 5 shows, for smaller

slag grains, chargeability is associated with lower relaxation times and for bigger slag grains, chargeability is associated with higher relaxation time (sample 13), but the grain size is in limited range. Figure 6 shows, for smaller slag grains, most of the chargeability is associated with lower relaxation times and for bigger slag grains, most of the chargeability is associated with higher relaxation time (sample 13) (Nordsiek and Weller, 2008).

Table 2: Comparison of SIP results derived from non-negative least square algorithm and that of Nordsiek and Weller (2008) shown along the permeability values.

Sam-ple no.	Published parameter-rs (Nordsiek and Weller, 2008)		Calculated parameters using equations (9) and (10)		Error between published and calculated parameters (%)		Permeability (mD) using equations (13) and (14)
	m_t	$\bar{\tau}$ (10^{-3} s)	m_t	$\bar{\tau}$ (10^{-3} s)	m_t	$\bar{\tau}$ (10^{-3} s)	
6	0.160	17.232	0.1603	17.700	0.188	2.844	6.0237×10^{-10}
13	0.358	842.766	0.3493	858.400	2.458	0.332	8.7140×10^{-8}

Quality check by synthetic example

To check the quality of non-negative least square algorithm, I have taken $A_{15 \times 10}$ matrix (**over-determined system**).

$$A = \begin{pmatrix} 3 & 2 & 0.4 & 0.5 & 0.6 & 0.1 & \dots & 0.9 \\ 1 & 4 & 3 & 0.5 & 0.3 & 0.1 & \dots & 0.8 \\ 0.5 & 2 & 5 & 4 & 0.6 & 0.3 & \dots & 0.9 \\ 0.4 & 0.1 & 3 & 6 & 5 & 0.4 & \dots & 0.8 \\ 0.3 & 0.2 & 0.6 & 4 & 7 & 6 & \dots & 0.1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0.1 & 0.4 & 0.2 & 0.5 & 0.7 & 0.8 & \dots & 0.8 \end{pmatrix},$$

$$x_{\text{true}} = \begin{pmatrix} 1 \\ 2 \\ 1.5 \\ 3 \\ 2.5 \\ 4 \\ 1 \\ 3.5 \\ 4 \\ 2.5 \end{pmatrix},$$



Estimation of Chargeability and Permeability using Spectral Induced Polarization Data



Solve Ax and calculate the value of b matrix

$$b = Ax_{true} = \begin{pmatrix} 14.60 \\ 23.00 \\ 33.70 \\ 43.05 \\ \vdots \\ 32.40 \end{pmatrix},$$

Now suppose we don't know the value of x_{true} , and then we solve the equation $Ax=b$ for x with the help of Non-negative least squares algorithm (x_{NNLS}).

$$x_{NNLS} = \begin{pmatrix} 1 \\ 2 \\ 1.5 \\ 3 \\ 2.5 \\ 4 \\ 1 \\ 3.5 \\ 4 \\ 2.5 \end{pmatrix}$$

The value of x_{NNLS} is well matched with the given x_{true} values. The rms error between x_{NNLS} and given x_{true} is 8.6950×10^{-15} . The rms error is very less, so the quality of our result is good.

Conclusions

Published spectral IP data have been used to derive the amplitude and phase spectra. The chargeability is calculated by the Debye Decomposition in frequency-domain by non-negative least square algorithm. The chargeability values in turn are used to calculate the integrated parameters and compared with published parameter (Nordsiek and Weller, 2008). Integration parameters like total chargeability etc. are estimated from SIP data closely match with that of published parameter (Nordsiek and Weller, 2008). Permeability is also calculated with the help of porosity and average relaxation time. The quality of non-negative least square algorithm is also checked by synthetic model, which indicates the good quality of result and the rms error is very less.

Appendix:

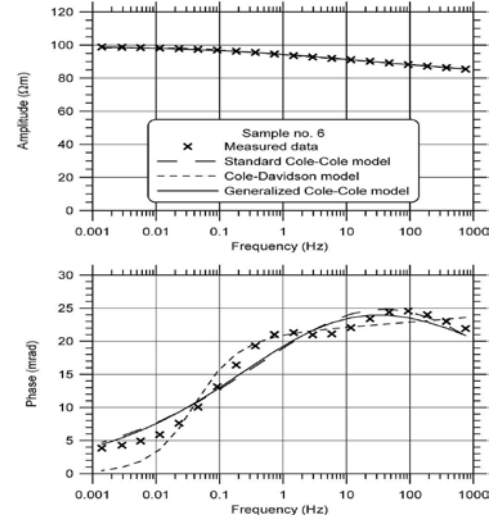


Figure 1: Measured spectra of sample number 6 and fits of Cole-Cole type models. a) Spectra of resistivity amplitude. b) Spectra of phase shift between current and voltage signal (Nordsiek and Weller, 2008).

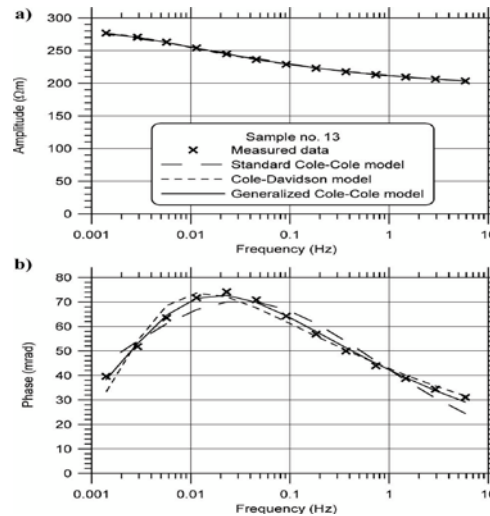


Figure 2: Measured spectra of sample no. 13 and fits of Cole-Cole type models. a) Spectra of resistivity amplitude. b) Spectra of phase shift between current and voltage signal (Nordsiek and Weller, 2008).



Estimation of Chargeability and Permeability using Spectral Induced Polarization Data



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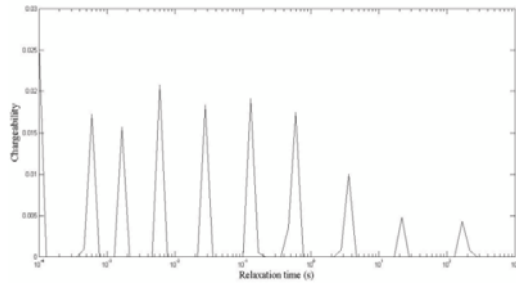


Figure 3: Chargeability plot versus relaxation time for sample number 6.

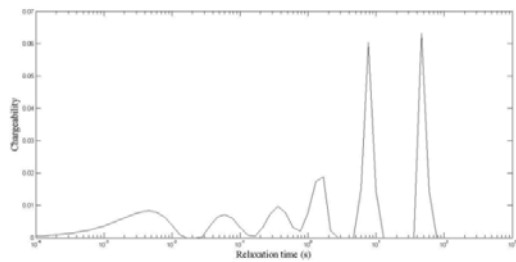


Figure 4: Chargeability plot versus relaxation time for sample number 13.

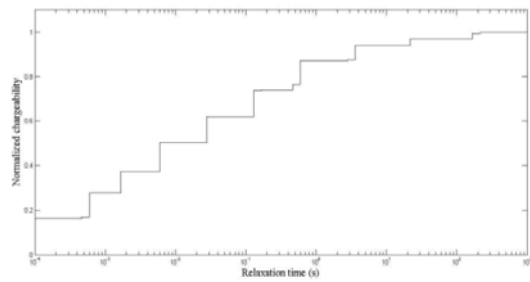


Figure 5: Normalized chargeability plotted versus relaxation time as a cumulative curves for sample number 6.

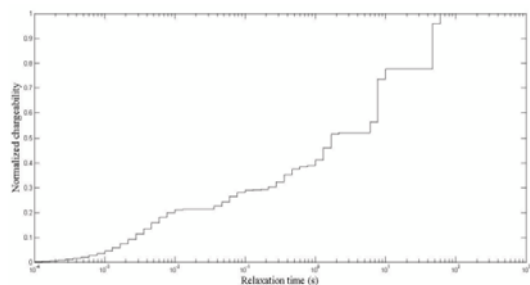


Figure 6: Normalized chargeability plotted versus relaxation time as a cumulative curves for sample number 13.

References

Cole, K. S., and R. H. Cole, 1941, Dispersion and absorption in dielectrics; *Journal of Chemical Physics*, 9, 341–351.

Davidson, D. W., and R. H. Cole, 1951, Dielectric relaxation in glycerol, propylene glycol and npropanol; *Journal of Chemical Physics*, 19, 1484–1490.

Knoll, M. D., R. J. Knight, and E. Brown, 1994, Can accurate estimates of permeability be obtained from measurements of dielectric properties?; *Proc. Symp. on the Application of Geophysics to Engineering and Environmental problems*, 25–35.

Morgan, F. D., and D. P. Lesmes, 1994, Inversion for dielectric relaxation spectra; *Journal of Chemical Physics*, 100, 671–681.

Nordsiek, S., and A. Weller, 2008, A new approach to fitting induced-polarization spectra; *Geophysics*, 73, F235-F245.

Reynolds, J. M., 1997, *An introduction to applied and environmental geophysics*; John Wiley & Sons, pp. 523-552.

Tarasov, A., and K. Titov, 2007, Relaxation time distribution from time domain induced polarization measurements; *Geophysical Journal International*, 170, 31–43.

Telford, W. M., L.P. Geldart, R.E. Sheriff, 1990, *Applied Geophysics*, 2nd ed.; Cambridge University Press, Cambridge, pp. 578-583.

Tong, M., W. N. Wang, L. Li, Y. Z. Jiang and D. Q. Shi, 2004. Estimation of permeability of shaly sand reservoir from induced polarization relaxation time spectra, *J. Petrol. Sci. Eng.*, 45, 1–10.

Tong, M., L. Li, W. Wang, and Y. Jiang, 2006, Determining capillary-pressure curve, pore-size distribution and permeability from induced polarization of shaly sand; *Geophysics*, 71, N33–N40.



Estimation of Chargeability and Permeability using Spectral Induced Polarization Data



Weller, A., 2003, Spektrale Induzierte Polarisation in der Archäometrie, in Freiburger Forschungshefte, Geophysik bei unterschiedlichen Skalierungen der geologischen Strukturen; TU Bergakademie Freiberg, C-496, 14–29.

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