9th Biennial International Conference
& Exposition on Petroleum Geophysics

"HYDERABAD 2012"

P-451

An evaluation of Fast sweeping method for Eikonal equation and its adjoint

***Puneet Saraswat**[1] and Mrinal K. Sen*[2]

Summary

Fast sweeping method (FSM) for Eikonal type equations provides an iterative method which incorporates Gauss- Seidel iteration and an upwind difference scheme with alternating sweeps for solving a discretized system. Unlike conventionally used algorithms such as finite difference, fast marching etc. each sweep in an FSM follows characteristics of the corresponding Eikonal equation in a certain direction, and is highly suitable for large data-sets. In this paper we incorporate the fast sweeping approach for solving Eikonal and adjoint equations to generate travel time maps and gradient of misfit function respectively. The importance of these types of mapping is realized in a number of applications such as traveltime tomography, full waveform inversion and velocity modeling. We present our evaluation with application to synthetic Marmousi velocity model data. We will employ the Eikonal based gradients to guide updates in full waveform inversion and migration velocity estimation.

***Keyword**: Fast sweeping method, Eikonal equation, adjoint state, gradient*

Introduction

The Eikonal equation is given by,

$$|\nabla T|^2 = S^2(x) \quad , \quad (1)$$

where S is the slowness. The Eikonal equation has many application in graphics, optimal control, planning, seismology etc; it describes travel time *T(x)* of a wave propagating with slowness S in space x. The approach followed to efficiently solve eikonal type equations is to treat the problem as stationary boundary value problem and designing an efficient algorithm to solve the discretized equation. A number of numerical algorithms have been proposed to solve eikonal type equations such as finite difference, fast marching etc. The advantage of fast sweeping method (FSM) over these methods is that FSM follows causality along the characteristics in parallel way which means various characteristics are divided into finite groups based on their direction and a Gauss Seidel type iteration solves these groups with alternate sweeps (Zhao 2004). This provides a fast and efficient method for solving eikonal equation and forward modeling for computing travel time map from the viscosity solution of eikonal equation from a given velocity model. Another important application of FSM is realized in calculating the gradient of the misfit function (equation 2)

$$F(s) = \frac{1}{2} \sum_{i=1}^{N_{shots}} \sum_{j=1}^{N_{receivers}} \left| T_{i,j}(s) - T_{i,j}^{obs} \right|^2 \qquad (2)$$

Forward modeling is used to compute the first arrival travel times T at each point x of the velocity model solving the Eikonal equation and its point source x_s condition. The adjoint state method then computes the gradient of the misfit function F (Eq. 2) with respect to the velocity model c (reciprocal of slowness S) with the following formula:

$$\nabla F(c) = \frac{1}{c^3} \sum_{i=1}^{N_{shots}} \lambda_i(c) \quad , \qquad (3)$$

where λ represents the adjoint state variable. Further details of the mathematics can be found in Sei and Symes (1994), and Leung and Qian (2006). The adjoint state variable is calculated for each source location and summed to get a global gradient. FSM provides a fast and effective way to generate λ (Taillandier et. al., 2009).

[1]Indian School of Mines, [2]University of Texas at Austin
Room 46 C sapphire hostel, Indian School of Mines,
puneet.agp@ismu.ac.in

Method

We implemented a FSM described in detail in Zhao (2005). In a complex velocity model, waves can propagate in any direction. The fast sweeping method incorporates a local as well a global scheme of sweeps using a non-linear upwind scheme in an alternating order. This makes it suitable for computing viscosity solution of Eikonal equation following the local characteristic of the velocity model. Below we briefly describe the essential steps in fast sweeping method for a 2D rectangular grid of size N_x X N_z.

- Initially assign travel time t(source)=0 and larger values at other grid location say infinity.
- The FSM method then updates values to t(i,j) following sweeps in negative X and Z directions (for 2D case).
- At each grid (i,j),minimum travel time is computed using Podvin & Lecomte's operators (Podvin & Lecomte, 1991).
- The travel time computed above is compared with the current value to travel time and minimum is retained.
- The above procedure is followed for four alternating sweeps for 2D data and eight for 3D (Zhao, 2005).

The above procedure is followed for each grid point several times until a minimum time is found for each, giving this method a optimal complexity of O(N) where N= N_x X N_z, making it fast and easy to implement. Practical implementation of fast sweeping method for forward modeling takes the velocity data as input which yields a first arrival travel time map over the entire domain of the velocity field. In this study we choose to describe velocity model in a rectangular grid pattern which is perhaps the most suitable parameterization of Eikonal solvers. The inverse calculation for velocity modeling and tomography problem involves solution of adjoint state equation with picked travel times and initial or guess velocity model as input. The inversion mainly relies on computation of gradient of misfit function. With proper gradient calculated, a simple steepest descent method can be incorporated such as conjugate gradients to minimize the misfit function. Figure 1(Taillandie et. al., 2007) shows the important steps towards forward modeling and calculation of global gradient. Corresponding to each shot, a travel time map is generated solving equation (1) with the velocity model as input to forward FSM solver. The residuals computed by subtracting observed data from computed travel times serve as initial condition to adjoint state solution and generation of local gradients corresponding to each shot. The local gradients thus obtained can be algebraically summed up to obtain global gradient. This is the most important step for all types of tomography problems. Figure 1(Taillendier et. al., 2007) illustrates the procedure described above towards the solution of eikonal and adjoint state equations. The advantage of FSM towards this approach is that the algorithm can be easily parallelized over large data-set minimizing memory requirements and decreasing computation time.

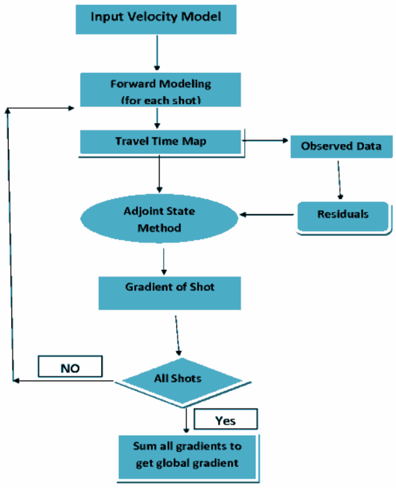

Figure 1: Flowchart for the solution of Eikonal and adjoint state equations

Examples

To illustrate our implementation of travel time mapping based on Eikonal equation followed by gradient of misfit function, we used the standard Marmousi velocity model. The model (Figure 2) consisted of 122 samples in z direction and 384 samples in x direction; the grid spacing between two adjacent cells was 24 m. Hence, the span of the velocity model was approximately 3 km in depth and 9 km in horizontal direction. Figure 1 shows the velocity model used in this study. The forward solution was carried out with fast sweeping method with a shot at 3.7 km. Figure 3 shows the travel time map generated using FSM from viscosity solution of the Eikonal equation (1).

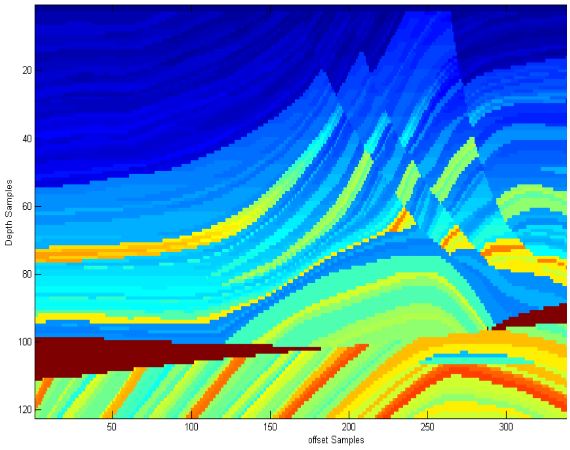

Figure 2: Marmousi velocity model

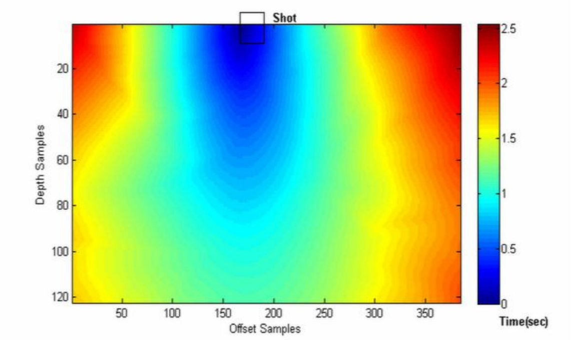

Figure 3: Travel time map from FSM.

Next, we used a smooth Marmousi velocity model (Figure 4) and its travel time map (Figure 5) for inverse problem and calculation of gradient of misfit function. The fast sweeping method is then set for solving the gradient of the misfit function (Equation 2) with the travel time difference for exact and smooth velocity model as the initial condition. To compute the gradient of the misfit function, first we need to initialize λ (the adjoint variable) at the surface using the time residuals which are first initialized on the surface of the model (Figure 6). The adjoint state method is essentially the back propagation method of adjoint variable to the source in current velocity model. For demonstration, we placed a receiver at 6 km and carried out adjoint calculation; the importance of adjoint state computation lies in generating gradient maps for each shot and summing up together to generate global gradient, which can easily be inverted using conjugate gradient. Figure 7 shows the plot of adjoint variable λ for single source receiver case. It clearly shows that the travel time from source to receiver is sensitive to velocities along the stationary phase path. It also shows a finite width and therefore, it can be used in finite frequency seismic inversion as a guide. To demonstrate the efficacy of the algorithm further, we placed four receivers on the surface of the model and shot is placed at the same location as mentioned above; the lambda values from source to receiver are very well mapped and are illustrated in Figure 8.

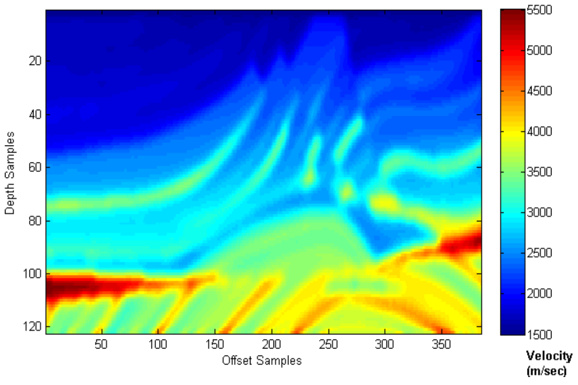

Figure 4: Smooth velocity model for adjoint state solution.

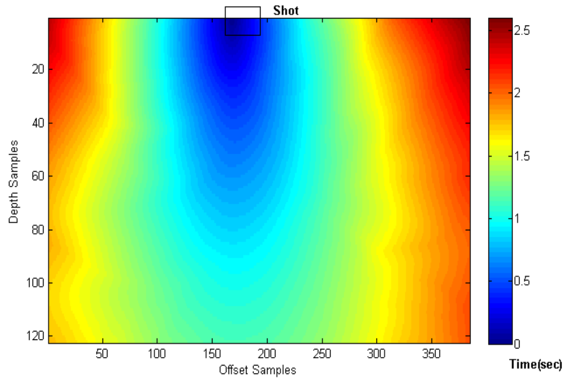

Figure 5: Travel time map for smooth model using FSM.

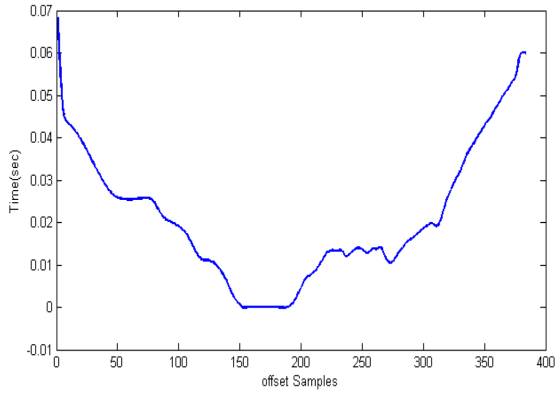

Figure 6: Time residuals to initialize λ on top of the model.

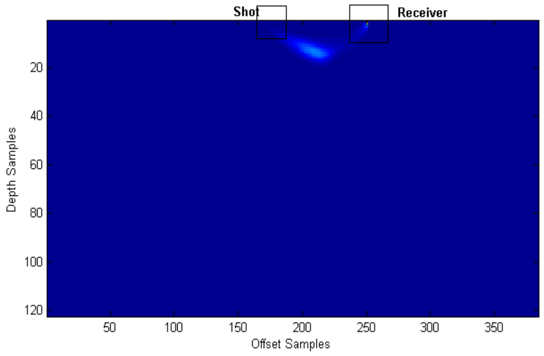

Figure 7: Gradient of misfit function for single source receiver pair.

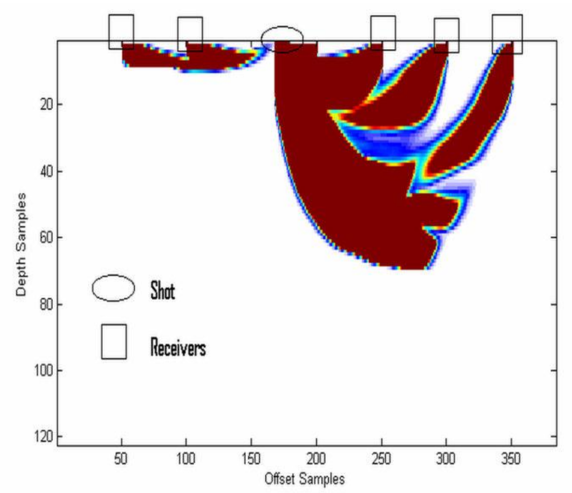

Figure 8: Map of adjoint variable for four receivers and single shot.

After computing λ, the gradient of misfit function is simply calculated by dividing λ by third power of velocity using the Equation shown below:

$$\frac{\partial L}{\partial \mathbf{c}} = -\int_{\Omega} \mathrm{d}x \frac{\lambda(x)}{\mathbf{c}^3(x)} = \frac{\partial J}{\partial \mathbf{c}} \quad (3)$$

When these gradient calculations are used in a local optimization scheme such as steepest descent or conjugate gradient, the inversion process can converge within few iterations; this process will be presented in our future studies.

Conclusions

The FSM method for solving Eikonal equation has recently been reported in literature. In this paper, we carried out an evaluation of the algorithm with application to a complex velocity model – the Marmousi model. The numerical experiment was conducted on a single desktop computer; the memory allocation needed being only few times the memory size of the velocity model. These algorithms make the crucial base for problems like tomography and full waveform inversion. The algorithm is also capable of handling data-set of any size and can be easily parallelized (Datta, Saraswat and Sen 2012). We will also present some results from anisotropic Eikonal equation and velocity analysis by a local optimization scheme.

References

Billette, F. and Brandsberg-Dahl, S., 2005, The 2004 BP Velocity Benchmark, 67th Meeting, EAGE Conference & Exhibition, Expanded Abstract, B035.

Datta, Leung, S. and Qian, J., 2006, An adjoint state method for three-dimensional transmission traveltime tomography using first arrivals, Communications in Mathematical Sciences, 4:249–266.

Paige, C.C. and Saunders, M.A., 1982a, LSQR: an algorithm for sparse linear equations and sparse least squares, ACM Transactions on Mathematical Software, 8, 43-71.

Paige, C.C. and Saunders, M.A., 1982b, LSQR: sparse linear equations and least squares problems, ACM Transactions on Mathematical Software, 8, 195-209.

Plessix, R. E., 2006, A review of the adjoint state method for computing the gradient of a functional with geophysical applications, Geophysical Journal International, Volume 167, Issue 2, 495-503.

Podvin, P. and Lecomte, I., 1991, Finite difference computation of travel times in very contrasted velocity models: a massively parallel approach and its associated tools. Geophysical Journal International, 105:271-284.

Sei, A. and Symes, W. W., 1994, Gradient calculation of the traveltime cost function without ray tracing, 65th Meeting, Society of Exploration Geophysicists, Expanded Abstracts, 1351-1354.

Tallinder C., M. Nobel, H. Chauris and H. Calandra, 2009, First arrival travel time tomography based on adjoint state method. Geophysics Vol. 74, pp. 57-66.

Zelt, C. A. and Barton, P. J., 1998, 3D seismic refraction tomography: A comparison of two methods applied to data from the Faeroe Basin, Journal of. Geophysical Research, 103:187-7210.

Zhao, H. K., 2005, Fast sweeping method for Eikonal equations, Mathematics of Computation, 74:603-627.