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On some critical aspects of numerical simulation of seismic wave propagation

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Summary

Numerical simulation of seismic waves is essential for understanding field observations and developing inversion schemes. The wavefields generated by the most popular numerical methods such as finite differences and finite elements are often contaminated with grid dispersion and edge reflections. We will present a grid dispersion and stability analysis based on a generalized eigenvalue formulation. Our analysis reveals that, for a spectral FE of order 4 or greater, the dispersion is less than 0.2% at 4-5 nodes per wave length and the scheme is isotropic. The FD and classical FE require a larger sampling ratio to obtain results with the same level of accuracy. The staggered grid FD is an efficient scheme but the dispersion is angle dependent with larger values along the grid axis. On the other hand, spectral FE of order 4 or greater is isotropic with small dispersion making it attractive for simulations for long propagation times. Further, we will discuss a new time-space domain finite difference scheme with adaptive length spatial operators. We also employ a simple scheme to absorb reflections from the model boundaries in numerical solutions of wave equations. This scheme divides the computational domain into boundary, transition, and inner areas. The wavefields within the inner and boundary areas are computed by the wave equation and the one-way wave equation, respectively. The wavefields within the transition area are determined by a weighted combination of the wavefields computed by the wave equation and the one-way wave equation to obtain a smooth variation from the inner area to the boundary via the transition zone.

***Keyword**: Seismic waves, finite elements, finite differences*

Introduction

Synthetic seismograms are an essential tool for interpretation and processing of seismic data. This involves analytical or numerical solution of elastic wave equation in heterogeneous media. Since analytic methods either obtain an approximate solution for realistic media or obtain exact solutions for simple media, pure numerical methods for solving elastic wave equation are becoming increasingly popular. There exist four distinct numerical methods for numerical simulation of wave equation, namely

- Finite difference method (FDM)
- Pseudo-spectral method (PSM)
- Finite volume method (FVM)
- Finite element method (FEM)

All of these methods use different approximation to evaluate the partial derivatives appearing in the wave equation. These methods are, in principle, capable of handling arbitrary laterally heterogeneous media but the reliability of the solution depends strongly on the order of approximation used. In this paper we will attempt to address the following critical questions related to numerical simulation of seismic wave propagation

- Time Domain or Frequency domain?
- First order velocity-stress or second order displacement formulation?
- FDM or FEM ?
- Accuracy: Grid dispersion and stability?
- Fixed vs Variable grid – fixed grid with variable operator length?
- Absorbing Boundary Condition (Absorbing)?

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Method

Basic Equations

Two fundamental equations, namely, the generalized Hooke's Law (Eq. 1) and equation of motion (Eq. 2) govern the state of time and space dependent deformation in an elastic solid.

$$\boldsymbol{\tau} = \lambda(\nabla\cdot\mathbf{u})\mathbf{I} + \mu\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right) \quad (1)$$

$$\rho\frac{\partial^2\mathbf{u}}{\partial t^2} = \nabla\cdot\boldsymbol{\tau} + \mathbf{f} \quad (2)$$

Where λ and μ are the Lame parameters, **u** is the displacement vector and τ is the fourth rank stress tensor. Substituting Eq. (2) into Eq. (1) we obtain the following wave equation

$$\rho\frac{\partial^2\mathbf{u}}{\partial t^2} = \nabla\cdot\left(\lambda(\nabla\cdot\mathbf{u})\mathbf{I} + \mu\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right)\right) + \mathbf{f} \quad (3)$$

From (1) and (2) we can derive a first order system of coupled velocity-stress system. Numerical methods estimate displacement (or particle velocity) and stress values as a function of space and time. We can take a Fourier transform of (3) and obtain a frequency domain version of the wave equation. Thus we have at least three choices: (1) use a first-order coupled system of equation for particle velocity and stress (2) a second order PDE for particle displacement, and (3) a first or a second order system in frequency domain. All of these forms have been used in various applications; they have their own pros and cons. Here I will review time domain formulations using FDM and FEM.

Finite Differences

In a finite difference scheme, the derivative operators are approximated by a series where the coefficients are computed using different levels of approximation. For example, a Taylor's series based approximation with 4th order terms in space derivatives and second order terms in time derivative, is very popular. Given the order of finite difference operator, the accuracy of the approximation is dictated by the grid spacing in space and time. The time domain formulation results in an explicit scheme. Given the initial wavefield, the wavefields can be marched in time in all spatial locations. Note that the derivative operators are approximated directly in the wave equation – this is called a strong formulation.

Finite Elements

The classical advantages of the FEM that motivated the early work include the flexibility with which it can accommodate surface topography, discontinuities in the subsurface model and boundary conditions, and the ability to approximate the wave field with polynomials of arbitrarily high degree. Unlike in the Finite Difference Methods (FDM) in which one simply uses Taylor series to approximate the derivatives in the wave equation, in the FEM we work with the so-called weak formulation of the wave equation and a representation of the solution as a linear combination of shape functions. This results in a linear system of equations, the solution of which involves costly inversions of large (albeit sparse) matrices rendering the implementation slow in comparison with the competing FDMs.

Fortunately, in the recent past there have been several developments that helped the FEM overcome the limitations and thus have made the method attractive once more. The main ideas in these developments are (1) the use of high-degree polynomials to approximate the wave fields, (2) the diagonalization of the matrices to be inverted (called mass matrices) through mass-lumping techniques and (3) the use of high-order time-stepping schemes. The methods that combine these ideas are the Spectral Element Method (SEM), and the Discontinuous Galerkin Method (DG).

Errors in FD and FE calucation

There are two serious problems that one must address in order to make use of FDM and FEM simulation of seismic wave propagation. They are: grid dispersion and stability and edge reflections.

Grid Dispersion and stability: Grid Dispersion is a numerical artifact that causes the higher frequency waves to travel at a different velocity than the lower frequency waves. Stability implies that the numerical solution remains bounded at any given time t. Grid Dispersion and Stability conditions are a basic tool in Seismic Modeling to determine the simulation parameters. Grid dispersion and stability analysis is generally straightforward but to carry out these analyses for SEM and DG is rather complicated due to the fact the node spacing is irregular. We have carried out rigorous grid dispersion analysis that is very general in that it includes standard and staggered grid finite differences, spectral elements and DG methods (DeBasabe and Sen 2007). We assume for the analysis that the medium is Isotropic, Homogeneous, Unbounded, Source free, and Acoustic or Elastic. These are the usual assumptions for this type of analysis. Furthermore, we make the following assumptions on the FEM discretization. The elements are square and periodic in each direction and the basis functions in higher dimensions are tensor products of the 1D basis functions. We derive the following results for SEM

- Continuous in time case

$$\text{P-wave dispersion: } \frac{\alpha_h}{\alpha} = \frac{1}{2\pi s}\sqrt{\Lambda_2}$$

$$\text{S-wave dispersion: } \frac{\beta_h}{\beta} = \frac{r}{2\pi s}\sqrt{\Lambda_1}$$

- Finite differences in time case

$$\text{P-wave dispersion: } \frac{\alpha_h}{\alpha} = \frac{1}{\pi s\, \mathrm{q}}\sin^{-1}\left(\frac{\mathrm{q}}{2}\sqrt{\Lambda_2}\right)$$

$$\text{S-wave dispersion: } \frac{\beta_h}{\beta} = \frac{r}{\pi s\, \mathrm{q}}\sin^{-1}\left(\frac{\mathrm{q}}{2}\sqrt{\Lambda_1}\right)$$

where Λ_1 and Λ_2 are the smallest eigenvalues, $\mathrm{q} = \alpha\Delta t/h$ is the stability parameter, $s = h/(\kappa L)$ is the average sampling ratio in the element and L is the wavelength.

An example of reduction of grid dispersion in the SEM case is given in Figure 1.

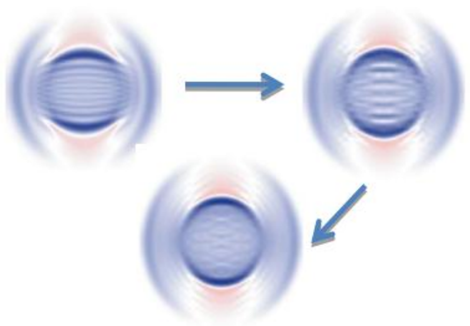

Figure 1: Display of a snapshot of wavefields in a homogeneous medium. As we follow the arrows, we satisfy grid dispersion criterion.

Similarly we have generated new finite difference coefficients that are essentially dispersion-free. These are time-space domain dispersion relation based schemes (Liu and Sen 2009). An example of computation using time-dispersion based FD calculation in 1D is shown in Figure 2.

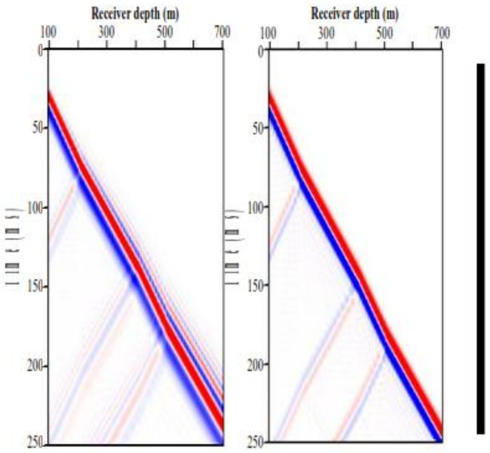

Figure 2. VSP simulation: the left panel was computed using standard finite difference scheme and the right hand panel was computed using a time-space domain dispersion free FD scheme. Note the marked improvement and reduced dispersion in the right hand panel (Liu and Sen 2009).

Edge reflection: In a numerical simulation of seismic wave propagation, the subsurface model is limited by the boundaries that limit the spatial extent of the model. The edges of the model, therefore, encounter discontinuous changes in the elastic parameters. These give rise to artificial reflections which often contaminate the reflections of interest. One obvious approach to reducing this is to enlarge the model extent so that the unwanted boundary reflections do not appear within the time window of interest. However, such an approach is not economic as it increases the model dimensions and therefore, the Computation and memory requirements increase substantially. Absorbing boundary conditions (ABCs) are commonly used to attenuate unwanted reflections from model boundaries introduced by a truncated computational domain. There are basically four kinds of ABCs. The first kind is a prediction based method. The boundary wavefield values are usually predicted by some approximations such as one-way wave equations (OWWEs). The second is an attenuation based method. Generally an exponential function is used to attenuate the wave field within a damping area near the boundary. The third is the perfectly matched layer (PML) method. This method is also an attenuation based method except that the attenuation operator is derived rigorously. The fourth type uses randomized velocities near the edges such that the reflections are incoherent. Of these three methods, the prediction based method has the least computational expense and it moderately absorbs boundary reflections. The attenuation based method has moderate computational expense but may perform the worst. The PML method is the most expensive computationally but generally obtains the best absorption.

We have developed a hybrid ABC by adding a transition area between the inner area and the boundary, and introducing a weighted strategy for predicted and modeled wavefields in the transition area (Liu and Sen 2010). This scheme has advantages of small computation cost and significant absorptions. In this paper, we compare the performance of hybrid ABC with that of the PML ABC for 2D high-order finite-difference (FD) acoustic modeling. An example of application of hybrid ABC is shown in Figure 3.

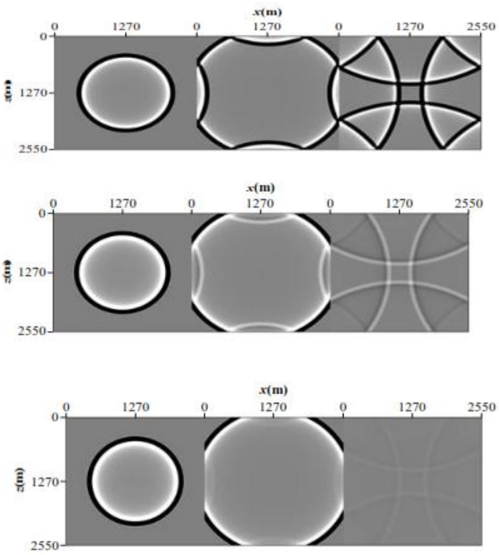

Figure 3. Snap shots for a homogeneous model at three different time steps from left to right: upper panel shows results with NO ABC; the edge reflections are very strong. The middle panel shows results with using prediction based ABC which still shows some edge reflctions. The lowest panel shows results from using our hybrid ABC; note that all the edge reflections have been absorbed. (Liu and Sen 2010).

We have also extended this method to 3D and to elastic media with standard and staggered grids. In all cases we are able to obtain excellent results.

Conclusion

Numerical simulation of seismic wavefields is essential for understanding seismic observation. It is even more important for inversion of seismic data which requires computation of adjoint wavefields using essentially the same numerical scheme. Their practical application is, however, hampered by the computational speed and memory requirements. Over the years, we have carried out several important improvements which make these methods fast and accurate enough for routine analysis of seismic data. This work is ongoing and our future developments will include anisotropy, stress simulation, hybrid FD-FE method and fracture simulation, to name a few.

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